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On the sensitivity of Apophis' 2029 Earth approach to small asteroid impacts

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ABSTRACT

Apophis' current trajectory takes it safely past our planet at a distance of several Earth radii on 2029 April 13. Here the possibility is considered that Apophis could collide with a small asteroid, like the ones that frequently and unpredictably strike Earth, and the resulting perturbation of its trajectory. The probability of an impact that could significantly displace Apophis relative to its keyholes is found to be less than 1 in 10^6, requiring a $\Delta v \gtrsim 0.3$mm/s, while for an impact that could significantly displace Apophis compared to its miss distance in 2029 it is less than 1 in 10^9, requiring a $\Delta v \gtrsim 5$cm/s. These probabilities are below the usual thresholds considered by asteroid impact warning systems.

Apophis is in the daytime sky and unobservable from mid-2021 to 2027. It will be challenging to determine from single night observations in 2027 if Apophis has moved on the target plane enough to enter a dangerous keyhole, as the deviation from the nominal ephemeris might be only a few tenths of an arcsecond. An impending Earth impact would, however, be signalled clearly in most cases by deviations of tens of arcseconds of Apophis from its nominal ephemeris in 2027. Thus most of the impact risk could be retired by a single observation of Apophis in 2027, though a minority of cases present some ambiguity and are discussed in more detail. Charts of the on-sky position of Apophis under different scenarios are presented for quick assessment by observers.

1. INTRODUCTION

Asteroid 99942 Apophis has been among the most heavily-studied near-Earth asteroids (NEAs) because of its much-anticipated close approach to Earth in 2029. Careful observation of this body since its discovery in 2004, and painstaking computation of its future path under the laws of physics show it missing our planet by a safe margin on 2029 April 13[1]. However even the best current projections do not account for the possibility of Apophis being struck by a small asteroid, such as regularly hit Earth and which are often widely visible in the form of meteors and fireballs. The impact of a small asteroid on Apophis —though very unlikely— would create a small impulsive change in the velocity of that asteroid. Such an asteroid strike on Apophis would be the natural analog of the kinetic impact process by which a $2.70 \pm 0.10 \times 10^{-3}$ m/s velocity change was imparted to asteroid Dimorphos by the Double Asteroid Redirection Test (DART) spacecraft in 2021 (Cheng et al. 2023).

In this work we will show that the probability of an impact sufficient to deflect Apophis in a dangerous manner is exceedingly low. However, the fact that Apophis' calculated path brings it near Earth at a distance of only 38012 ± 4 km (center-to-center) on 2029 April 13[2] drives us to consider even such highly improbable cases, to ensure our collective understanding of this asteroid's future motion is as complete as possible.

Asteroid collisions with other asteroids or comets are rare, but have been observed several times. Asteroid 596 Scheila, comet 354P/LINEAR and asteroid 493 Griseldis are all thought to have been struck by small asteroids (Ishiguro et al. 2011; Jewitt et al. 2011; Snodgrass et al. 2010; Jewitt et al. 2010; Tholen et al. 2015). Asteroid families are also believed to be the result of collisions (Michel et al. 2001).

[1] https://www.jpl.nasa.gov/news/nasa-analysis-earth-is-safe-from-asteroid-apophis-for-100-plus-years , retrieved 2024 April 15
[2] https://ssd.jpl.nasa.gov/tools/sbdb_lookup.html#/?sstr=99942, retrieved 2024 April 15

Wiegert & Hyatt (2024) showed that there are no expected collisions of Apophis with the catalogue of 1.2 million known asteroids, though there is some very low chance of collision with material accompanying a few of them. However, it is not known at this time whether the asteroids in question even have accompanying material. The possibility of an impact with material accompanying a known asteroid is implicitly included in our analysis here, but we are primarily concerned with the background population of small unseen meter-class asteroids and meteoroids that could hit Apophis at any time.

Though the calculated path of Apophis during its 2029 return is confidently expected to keep it from impacting our planet, the question of whether or not it will pass through a keyhole is a more complicated problem. A keyhole refers to a region of space near Earth which, if Apophis should pass through it during its 2029 close approach, indicates that it is on a collision course with Earth at a future date. Much work and attention was given to the keyhole problem, and in 2021, the problem was retired, at least for keyholes leading to impacts up to and including 2068, by a careful analysis from JPL[1]. This analysis is based on a careful reconstruction of Apophis' path from telescopic and radar observations. However it does not include the possibility of a random impulsive change in Apophis' velocity such as might occur from a collision with a small asteroid.

An additional element of the story is that Apophis has been largely unmonitored by telescopes since May 2021, and will remain so through 2027. This simply arises because of the relative geometry of Apophis, Earth and the Sun, which puts the asteroid in the daytime sky for the time span in question. Even professional telescopes cannot observe Apophis in broad daylight, in the same way that one cannot see the stars with the unaided eye during the day. It does not result from a defect in the eye or in the telescope, but simply from the overwhelming brightness of the daytime sky. Most space telescopes also avoid looking towards the Sun; the Hubble Space Telescope has a solar avoidance zone of 55 deg (Marinelli & Dressel 2024) and the James Webb Space Telescope, 85 deg (Nella et al. 2004). The Solar and Heliospheric Observatory (SOHO) can and does observe near the Sun, but its Large Angle Spectrometric Coronagraph (LASCO) has a limiting magnitude of $V = 9.5$ for solar system objects (Lamy et al. 2013) while Apophis is fainter than $V = 19$ until 2027. As a result, Apophis will remain essentially unmonitored for over six years. If it did suffer a significant impact during this time we might be unaware of the fact, and only able to infer that a perturbation occurred from observations taken once it returns to observability in 2027.

The purpose of this paper is to model the effect of a small asteroid impact on Apophis occurring after observations of it were forced to cease, in order to determine:

1. What magnitude of velocity change would be needed to significantly affect the details of Apophis' 2029 close approach? That is, what impulse could move Apophis on the target plane by an amount that could direct it into a keyhole to a collision at a future date, or to collision with the Earth or Moon in 2029?

2. What size asteroid would need to strike Apophis to create such an impulse, and what is the likelihood of such a collision occurring?

3. If Apophis suffered such an impact, how would the on-sky position of Apophis differ from its nominal ephemeris when it re-emerges to visibility in 2027? If Apophis' trajectory is found to differ from that expected, how can we quickly identify dangerous trajectories?

The probability of a small asteroid colliding with any asteroid over the course of a few years or decades is tiny and indeed could be considered negligibly small in almost any other case. It is only this asteroid's unusual trajectory and the attendant risk it poses to our planet that drives us to examine this scenario in detail here.

2. METHODS

Time frame of the study—We begin our study at the earliest time at which Apophis could have had an undetected impact, set by the last observations of Apophis recorded before it moved too close to the Sun on the sky. These last observations represent the final times when the presence of dust from a hypervelocity impact onto Apophis could have been detected telescopically. At this writing, the Minor Planet Center shows the last few observations of Apophis occur in May 2021 through April 2022[3]. The last observation from the major NEA surveys was 2021 May 12 for both PanSTARRS 1 Haleakala and Steward Observatory, Kitt Peak-Spacewatch, observations taken at a solar elongation angle of 63 degrees (Minor Planet Center 2021a). Observatory B18 - Terskol in Ukraine does report a additional

[3] https://minorplanetcenter.net/db_search/show_object?object_id=Apophis, retrieved 2023 April 13

position on 2021 May 20 at an elongation of 53 degrees (Minor Planet Center 2021b) and there is an occultation observation listed in April 2022 (Minor Planet Center 2024).

The major NEA surveys consistently provide the most reliable observations of near-Earth objects. And given that the DART impact into Dimorphos caused a noticeable dust cloud and brightening within minutes to hours after the event (Kareta et al. 2023), we can expect that a sizable impact on Apophis would be almost immediately detected by the major surveys if observing conditions were good.

The observations taken of Apophis after those of the major surveys represent important contributions to the monitoring of Apophis, but it is unclear whether they would have detected dust near Apophis under difficult near-Sun viewing. Occultation observations, often collected by amateur astronomers under challenging conditions, represent valuable high-quality measurements of Apophis' on-sky position but are not optimized for dust detection. As a result, we will adopt the date after the final observation by the major NEA surveys, 2021 May 13, as the first date at which an unseen impact might have occurred. We will see that earlier impulses do as a rule produce larger deviations from Apophis' expected trajectory, but that our results are not very sensitive to the precise timing of the impulse.

Based on the final recorded observations of Apophis, we will also adopt a solar elongation of 60 degrees as the practical though not absolute limit at which Apophis observations can be made. We will use this value to determine when Apophis can be re-observed again when it re-emerges from the daytime sky. We note that Apophis returned to solar elongations greater than 60 degrees during October 2021 to April 2022 at $m_V \approx 21$ but its solar elongation did not exceed 80 degrees. During this time, only the occultation observation mentioned earlier is recorded by the Minor Planet Center[3].

Model—We compute the path of Apophis within a model solar system, described below. To examine the effects of an asteroid impact on Apophis, we create a number of hypothetical versions of Apophis ("clones"). Each clone has the same initial conditions as Apophis at the start of the simulation, but each clone has a single velocity impulse applied to it. The impulse has a fixed magnitude Δv which we assume is uniformly distributed on the sphere, though in reality some directional asymmetry is expected (Le Feuvre & Wieczorek 2008; Robertson et al. 2021). In practice, the impulse direction is chosen by selecting three random numbers (R_x, R_y, R_z) uniformly within the interval (-1, +1). If $R_x^2 + R_y^2 + R_z^2 > 1$, the numbers are discarded and new ones drawn. If $R_x^2 + R_y^2 + R_z^2 <= 1$, then ($R_x$, R_y, R_z) is normalized and defines impulse direction of Δv in heliocentric Cartesian coordinates. This process ensures the impulses are indeed uniformly distributed on the sphere. The impulse occurs at a time chosen at a uniformly random point within the time frame examined. This time frame extends from 2021 May 13 (JD 2459347.5) through to Apophis' next close approach on 2029 April 13 (JD 2462239.5).

Apophis and its clones are modelled within a solar system which includes the eight planets and the Moon. Planetary initial conditions are from the JPL DE405 ephemeris (Standish 1998). All particles are integrated with the RADAU (Everhart 1979) algorithm with a tolerance of 10^{-12}. Post-Newtonian effects are included, as is the Yarkovsky effect with parameters as determined by JPL and which are assumed to be unchanged by the small asteroid impact. Apophis and its clones are treated as massless test particles.

Model tests—For testing purposes, we propagated our model forward eight years from our initial conditions to the close approach to Earth on 2029 April 13. Our model reports a close approach distance of 38105 km, 93 km ($< 10^{-8}$ au) from JPL CNEOS's current value of 38012 ± 4 km[2], and occurring within 10 seconds of the time reported by JPL. Our model therefore matches the JPL values closely. JPL does not report the target b-plane position associated with their current orbital solution for Apophis, but our b-plane error is certainly similar to that in the close approach distance, about 100 km. We will see that this is sufficient to resolve changes of interest (which will be at their smallest about 200 km on the b-plane, see Section 3.1). Our uncertainties are somewhat larger than JPL's but sufficiently accurate for the sensitivity study presented in this paper.

3. RESULTS AND DISCUSSION

3.1. *Sensitivity to impulses*

Could the impulsive change to Apophis' velocity caused by a small asteroid impact push Apophis into a keyhole? The keyhole analysis of Farnocchia et al. (2013) reveals that the pattern of keyholes on the target plane is very complex. Our goal here is not to determine the circumstances that would put Apophis into a specific keyhole. Rather we simply ask what size impulse would be needed to move Apophis substantially with respect to the most important keyholes, and thus require a possible re-evaluation of its future impact probability.

Farnocchia et al. (2013) reveals that Apophis would have to move approximately 200 km vertically on the target plane to reach the nearest important keyholes, and about 1500 km vertically on the target plane to reach the complex of secondary keyholes surrounding the 2036 primary resonant return. We will adopt these numbers as representative of the target plane displacements of interest.

From our simulations we find that a Δv of 3×10^{-4} m/s is sufficient to move Apophis 200 km on the b-plane to the nearest keyholes, and 3×10^{-3} m/s to move it 1500 km on the b-plane to the more distant 2036 keyhole complex (see Fig 1). Asteroid impacts are found to move Apophis efficiently vertically on the target plane; however, any particular impulse is as likely to move Apophis away from a keyhole as towards it, depending on the direction in which it was applied. A plot of the median $\Delta b/\Delta v$ over time in our simulations is presented in Figure 2. We conclude that a $\Delta v \sim 3 \times 10^{-4}$ m/s is the minimum impulse of concern, that is, it is the minimum necessary to deflect Apophis to one of the nearby keyholes. Impacts occurring later in time are less effective in moving Apophis on the target plane. Even if such an impulse should occur, the results would most likely be harmless. Nonetheless, monitoring Apophis telescopically for signs of such an event at the earliest possible date is recommended (see Section 3.3).

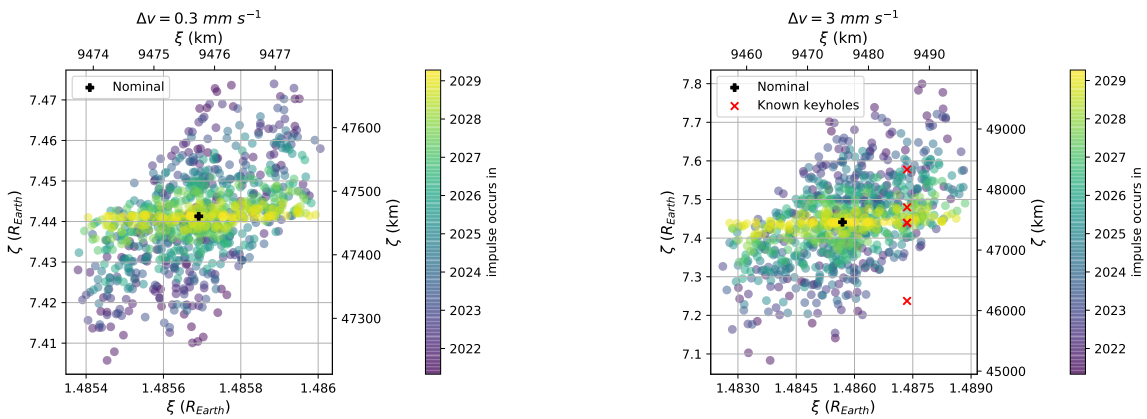

Figure 1. The Opik target plane under $\Delta v = 3 \times 10^{-4}$ m/s (left) and 3×10^{-3} m/s (right) impulses. Impulses of this order are able to produce displacements of Apophis on the target plane of $\sim$ 200 km and $\sim$ 1500 km respectively in the vertical direction, necessary to move the asteroid into keyholes. However, only very specific (and hence very unlikely) impulses will result in keyhole entry. Some important keyholes identified by Farnocchia et al. (2013) are indicated.

Our simulations show that small asteroid impacts move Apophis efficiently in the vertical (ζ) direction on the target plane. Since the encounter geometry of Apophis with Earth is such that a sufficient vertical displacement on the target plane would bring about an impact with our planet, we examine this possibility here as well. An impulse Δv of 5×10^{-2} m/s is found to be the minimum value that could result in impact with Earth in 2029, though the impact would have to be applied at a very specific time and direction for this to occur (see Fig 3). At $\Delta v \gtrsim 10^{-1}$ m/s, a wider range of impulse directions and timings can produce an Earth impact. An animation of the scenario with $\Delta v = 10$ cm/s is shown in Figure 4. For impulses in the 0.1-1 m/s range (the largest impulses considered here), the resulting probability of Earth impact hovers near 5%, as only certain impulse directions can direct Apophis to our planet, and larger impulses eventually start to miss Earth by passing on the opposite side (see Fig 3).

Small asteroid impacts cannot easily push Apophis into a collision with the Moon in 2029. The nominal close approach distance of Apophis to the Moon is about 96,000km (55 R_{Moon}). None of the cases examined here approach the Moon more closely than 40 R_{Moon}, all remain outside the Moon's sphere of influence (37 R_{Moon}).

Having determined the magnitudes Δv of the impulses of interest, we now turn to an analysis of the probability of an asteroid of the necessary size striking Apophis.

3.2. *Probability of small asteroid impacts on Apophis*

What size asteroid would need to hit Apophis in order to generate the impulses described earlier? To an order of magnitude, the change in velocity Δv in Apophis due to an asteroid impact scales by the impactor's relative momentum

$$\beta m v \sim M \Delta v$$

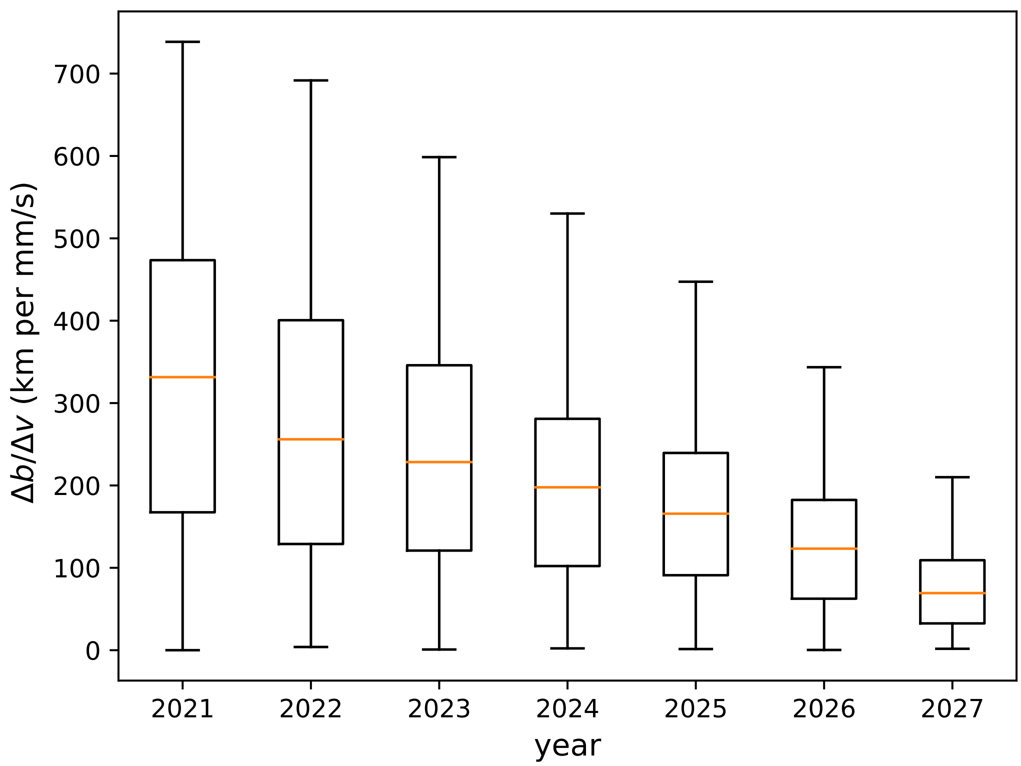

Figure 2. A box plot of $\Delta b/\Delta v$ over time. The boxes show the inner quartiles with a line at the median, while the whiskers extend across the full range of values. Impacts that occur earlier in time are more effective at moving Apophis on the target plane.

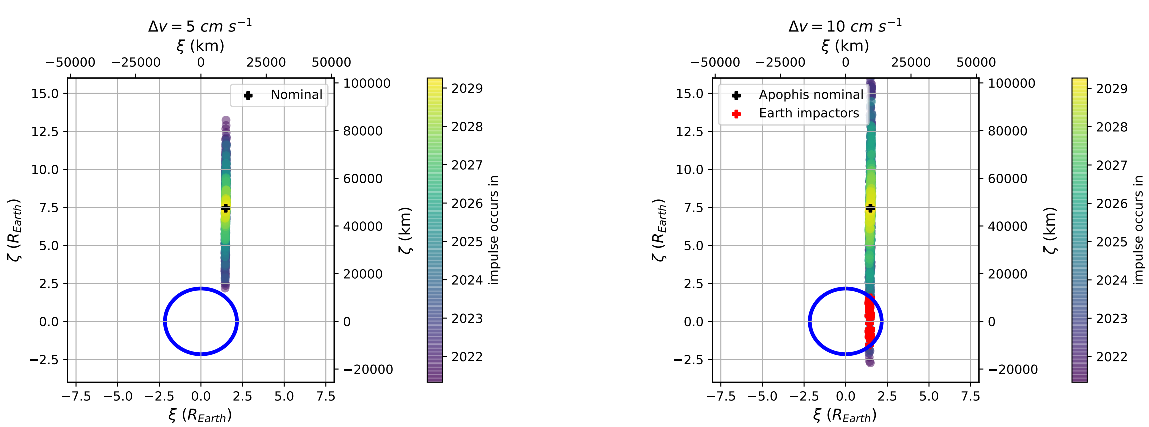

Figure 3. The Opik target plane under 5×10^{-2} m/s (left) and 10^{-1} m/s (right) impulses. The impact cross-section of the Earth in this case is 2.16 Earth radii (Farnocchia et al. 2013), and is indicated by the blue circle. Any point inside the circle represents a collision with our planet (red crosses). An impulse of 5×10^{-2} m/s could just reach Earth if applied early and directed in just the right direction. Impulses of $\Delta v \gtrsim 10^{-1}$ m/s may reach Earth over a wider range of impulse directions and timings.

$$\Delta v \sim \frac{\beta m v}{M} \tag{1}$$

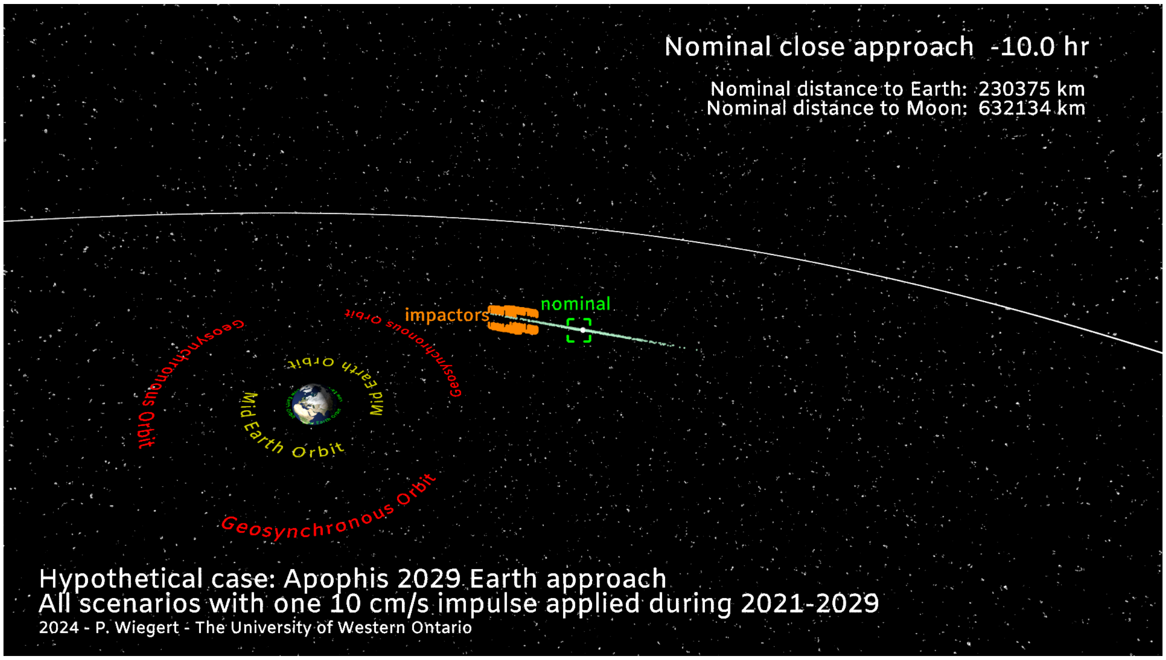

Figure 4. Illustration of the cases where one impulse $\Delta v = 10^{-1}$ m/s from a small asteroid impact has been applied to Apophis. The nominal (unperturbed) case is highlighted by a green reticle. Cases leading to Earth impact are indicated by an orange reticle. This figure is a single frame of an animation showing the approach of the asteroid and clones to Earth. The animation is available in the online journal, or at https://youtu.be/9UYM6MUqvYM. The animation is approximately 20 seconds long, and shows the relative positions of the Earth, Moon, Apophis and the clones during several hours around the 2029 close approach.

where M is Apophis' mass, $m \ll M$ is the impactor mass, v is the speed of the impactor relative to the asteroid, and β is an enhancement factor resulting from momentum carried away by ejecta. For context we consider the measured Δv of the DART impact into Dimorphos. Dimorphos has a 151 m diameter and a mass $M = 4.3 \times 10^9$ kg assuming a similar density to Didymos, while the DART impactor had a mass of 580 kg, hit at a relative speed of 6.1 km/s and had a measured β between 2 and 5 (Daly et al. 2023). Adopting $\beta = 3$, Eqn 1 yields $\Delta v = 2.4 \times 10^{-3}$m/s, while the measured velocity change of the DART impact was of $2.70 \pm 0.10 \times 10^{-3}$ m/s (Cheng et al. 2023). Cheng et al. (2023) find a mean value of $\beta = 3.6$ which yields $\Delta v = 2.88 \times 10^{-3}$m/s. These close matches reveal that Eq 1 is sufficient for our purposes.

The diameter d needed for an impacting asteroid to create a particular Δv in Apophis can be found from Eq 1, assuming roughly spherical shape and density ρ, as

$$d \sim 9.2 \left(\frac{\Delta v}{1 \text{ m/s}}\right)^{\frac{1}{3}} \left(\frac{\beta}{3}\right)^{-\frac{1}{3}} \left(\frac{\rho}{3500 \text{ kg m}^3}\right)^{-1/3} \left(\frac{v}{17 \text{ km/s}}\right)^{-\frac{1}{3}} \left(\frac{D}{340 \text{ m}}\right) \text{m} \tag{2}$$

where $D = 340$ m is Apophis' diameter (Brozović et al. 2018), and its density is taken to be 3500 kg m^3 (Binzel et al. 2009). The average impact velocity of bolides arriving at Earth $v \approx 20$ km/s (Grün et al. 1985; Brown et al. 2002; Greenstreet et al. 2012; Drolshagen et al. 2020) which includes a 11 km/s component due to Earth's gravitational attraction which will not apply at Apophis, so we adopt $v \approx \sqrt{20^2 - 11^2} \approx 17$ km/s for the typical relative velocity.

Equation 2 reveals that an impact able to move Apophis significantly relative to the 2029 keyholes ($\Delta v \gtrsim 3 \times 10^{-4}$m/s) would necessitate an impactor with $d \gtrsim 0.6$ m. In order to create the possibility of an impact with Earth itself in 2029, a $\Delta v \gtrsim 5 \times 10^{-2}$ m/s is needed, translating to $d \gtrsim 3.4$ m.

What are the probabilities of asteroids of sufficient size striking Apophis? Adopting the measured flux of meter-sized objects at Earth from Brown et al. (2002) we see that approximately 140 0.6-m diameter or larger objects, and only a single 3.4 m or larger object, strike the Earth each year on average. However, Apophis's physical cross-section is only $(0.17 \text{ km}/6378 \text{ km})^2 \approx 7 \times 10^{-10}$ that of Earth, so the rate of impacts is reduced by this factor. This puts the odds of Apophis being struck by an asteroid large enough to deflect it significantly relative to the 2029 keyholes is less than 1 in one million. Given that not all impulses of a given size move Apophis towards a keyhole, the odds of a small asteroid impact deflecting Apophis to a dangerous post-2029 encounter over the 8 year time frame examined here is minuscule. The odds of an unseen small asteroid deflecting Apophis enough to direct it into a collision with Earth in 2029 ($d \gtrsim 3.4$m, $\Delta v > 5 \times 10^{-2}$ m/s) is approximately 10^{-8}. Given that only 5% of such impulses are in the correct direction to generate an Earth impact (see 3.1), the overall probability of a small impact directing Apophis into collision with the Earth is less than one in 2 billion.

Our results depend on the densities ρ and velocities v of the small impactors at a given size. If either of these is higher than we assumed here, then the resulting Δv at a given diameter is also increased, and smaller impactors could create the necessary impulses. This could increase the probability of the deflections considered here occurring, since smaller impactors are more abundant. However, Eqn 2 depends on v and ρ only to the negative one-third power, and so the probabilities discussed here are not very sensitive to the details of the impactor population.

3.3. *On-sky position of Apophis*

We showed that the probability of a small asteroid deflecting Apophis substantially is exceedingly small. For any other asteroid, the eventuality might be dismissed as negligible. However given the disproportionate risks associated with Apophis, we turn to an examination of how and when we could determine whether such a deflection has occurred.

Apophis is as of this writing in the daytime sky, and cannot be monitored telescopically. If Apophis suffers an impact before it returns to visibility in 2027, the most easily detectable after-effects (i.e. optical brightening due the impact-produced dust cloud) could have dissipated. If that is the case, the effect of a small asteroid impact would perhaps only be revealed to telescopes on Earth by a deviation of Apophis from its expected position in the sky. Changes in Apophis' expected magnitude or even its distance could also be useful; we'll discuss those later in Section 3.3.3.

The detailed determination of an asteroid's trajectory requires large numbers of careful telescopic observations taken over a period of weeks, months or even years. Though this process will certainly be initiated by astronomers around the globe as soon as Apophis reappears, it is very hard to predict how long a full computation of Apophis' path based solely on post-2027 observations will take. Instead, we ask a simpler question here: if Apophis re-emerges into the night time sky on a trajectory perturbed enough to move it into an important keyhole or even to a 2029 Earth impact, how would its on-sky position differ from its current nominal ephemeris? What observations could quickly reveal whether it has been significantly perturbed?

For Earth-based observers, Apophis will emerge —barely— from the solar glare in late February 2027, after having been largely unobservable for over six years (Figure 5). The elongation of Apophis from the Sun reaches 60 degrees as seen from Earth's geocenter on 2027 February 22 (JD 2461458.5)[4]. At this point, the asteroid has an expected apparent magnitude of 21.0. But Apophis does not reach elongations above 62 degrees, hovering near 60 degrees until April 2027 before moving further into the daytime sky again. Apophis finally re-emerges to elongations greater than 60 degrees on 2027 December 6 (JD 2461745.5) at magnitude 19.7, remaining visible until 2028 June 30 (2461952.5). These time ranges will be used here to define "early" and "late" observing windows for Apophis.

What deviation from the nominal ephemeris would be detectable? For a single night's observation from a single station, we will consider that a displacement of more than 1 arcsec on the sky from the nominal RA or Dec will be detectable by ground-based telescopes, as this level is routinely achieved at professional observatories. On-sky positions can be measured to better than 1 arcsecond under good conditions, but given that we are considering observations of Apophis taken near the Sun, with higher than usual scattered light and potentially fewer visible stars for astrometric solutions, it is a reasonable if conservative assumption.

In Section 3.1, we saw that Farnocchia et al. (2013) determined Apophis will intersect the target b-plane 200-1500 km from the most important keyholes. As a result, we will take those particles found to move on the b-plane by $\Delta b > 200$ km to represent those which move appreciably with respect to the keyholes. These cases do not necessarily represent particles which have been perturbed into a dangerous keyhole. In fact the odds are very low that keyholes will be entered even under a large perturbation, but such a determination will require a full orbital analysis after Apophis reappears in 2027. Our goal here is simply to provide a framework in which cases of concern can be quickly identified for further study.

3.3.1. *Keyholes*

[4] The observational circumstances of Apophis reported here are from the JPL Solar System Dynamics Group, NASA/JPL Horizons On-Line Ephemeris System (Giorgini et al 1996) https://ssd.jpl.nasa.gov/horizons, retrieved 2024 March 17

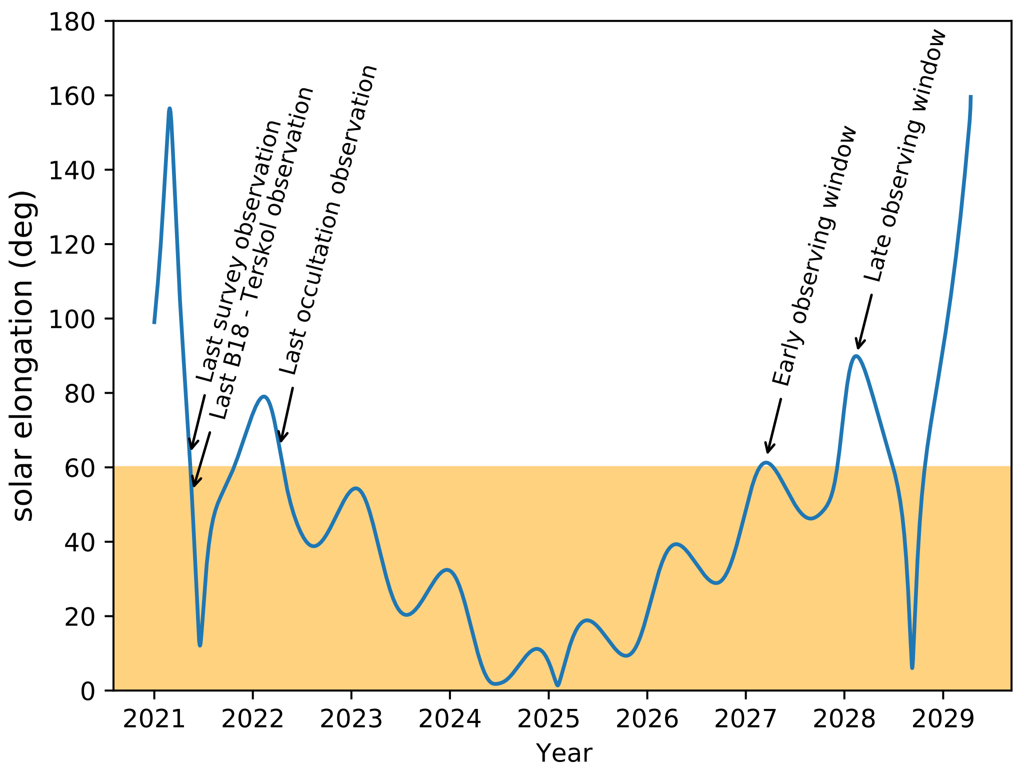

Figure 5. The solar elongation of Apophis as seen from Earth during 2021 to 2029. The start of each year is labelled on the x-axis.

In Section 3.1 we saw that a Δv between 3×10^{-4} and 3×10^{-3} m/s could move Apophis appreciably relative to keyholes. The effect of the larger of these Δv on the on-sky position of Apophis is shown in Figure 6.

Figure 6 illustrates that, if a 1 arcsecond displacement of Apophis is the observational limit, then many scenarios involving an impulse large enough to move Apophis appreciably on the target plane might not be detectable from a single-night observation during either the early or late observing windows. But displacement $\Delta\theta$ in on-sky position is not an unambiguous measure of Δb. To quantify this more carefully, we ask: if Apophis is observed to have some displacement $\Delta\theta$ from its nominal position, what is the probability that this case is associated with substantial movement on the target plane?

Figure 7 shows the probability of an observed on-sky offset $\Delta\theta$ being associated with $\Delta b < 200$ km, based on all the clones simulated in this work. Each clone is weighted by the probability of a small asteroid impact creating its associated Δv, using Eqn 1 to determine the asteroid size d needed, then assuming a small asteroid flux onto Apophis proportional to that given by Brown et al. (2002) for Earth (Section 3.2). Though this procedure is only approximate, it provides a useful probability-weighted outlook at possible outcomes. For example, if a deviation of Apophis from its nominal ephemeris position of $\Delta\theta \approx 0.1''$ was measured on 2027 April 12, Figure 7 reveals that the odds are about 50:50 that the case involves substantial motion of Apophis relative to the keyholes.

From Figure 7 we conclude that single night observations good to 1 arcsecond will be insufficient to determine if Apophis has moved relative to the important keyholes on the target plane. However, if Apophis is seen to be displaced by more than 0.2" from its ephemeris position at this time, then a substantial displacement of its intersection with the target plane has possibly occurred. This would not, however, mean that Apophis is on track to enter a dangerous keyhole; the odds are still in favor of a continued safe trajectory. But a displacement of Apophis of more than a few tenths of an arcsecond from nominal in 2027 would be cause for additional follow-up observations and analysis.

3.3.2. *Earth impact*

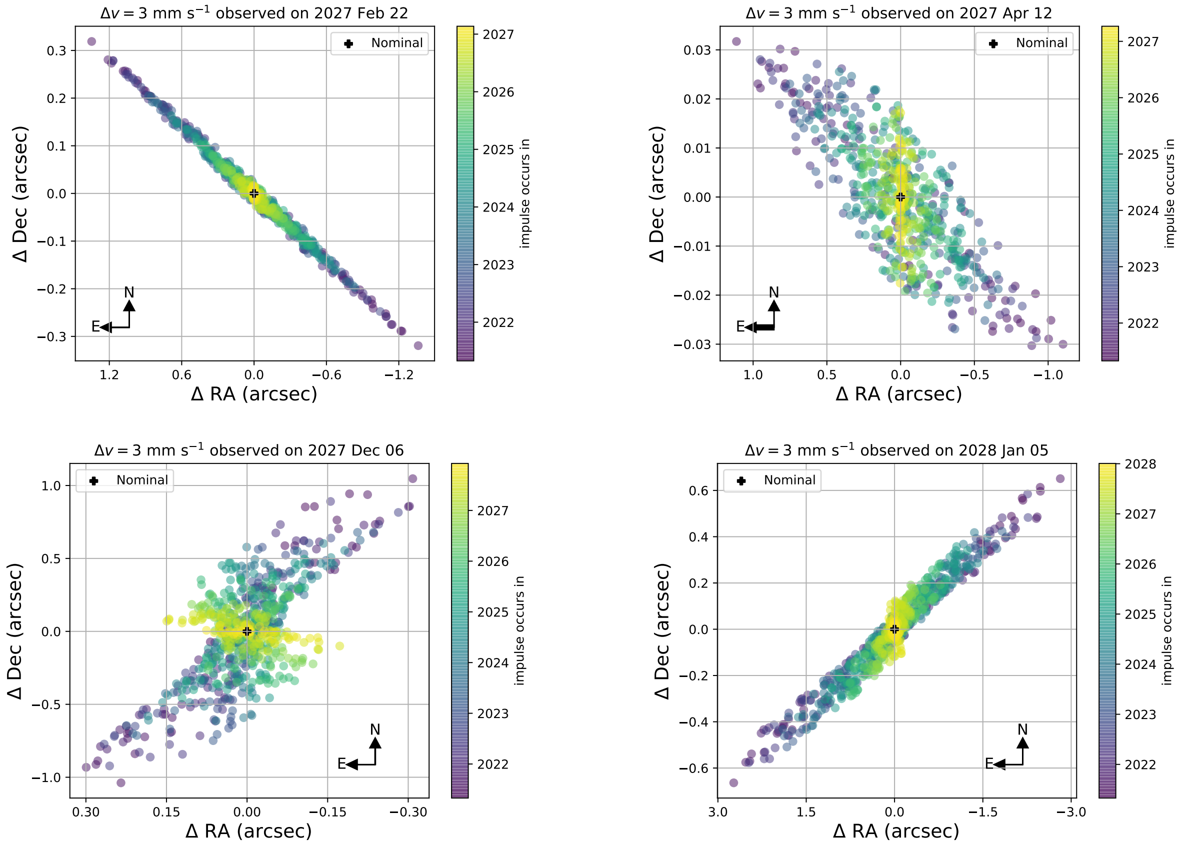

Figure 6. The on-sky displacement from the nominal Apophis ephemeris at the start (top left) and end (top right) of the early observing window, and the start (bottom left) and 30 days into (bottom right) of the late observing window, for an impulse $\Delta v = 3 \times 10^{-3}$m/s. Displacements are in arcseconds on the sky. The ephemeris uncertainty in Apophis' position is less than 0.01 arcseconds in all cases, smaller than the size of the plotting symbols.

The probability of a small asteroid deflecting Apophis onto an Earth-intersecting orbit in 2029 is exceptionally low, less than 1 in a billion by the analysis of Section 3.2. But if this unlikely event occurs, how will it manifest itself observationally when Apophis first returns to visibility in 2027?

Most cases leading to an Earth impact show easily measurable on-sky position differences from the nominal ephemeris during the early observing window; these are shown in Figure 8. Earth impactors form a distinct subset; however, positions within the 'red zone' of Figure 8 are not sufficient to identify Earth impactors unequivocally, as they overlap with other cases that have been heavily perturbed, but will not reach our planet.

Though most Earth impactors are significantly displaced from the nominal on-sky position of Apophis, not all are. Roughly 1% of Earth impactors appear less than 1 arcsecond from the nominal location on the sky when the early window opens, increasing to 3% at its closing. These mostly correspond to cases where the collision that created the velocity change happened within the 30 days prior to the date of observation: Apophis has not yet had time to move much from its nominal trajectory. Since the dust cloud and/or tail created by the DART impact was visible for at least 30 days after that event (Kareta et al. 2023; Opitom et al. 2023), we expect that the unlikely case of Apophis being both near its nominal on-sky position and on an Earth-intercepting trajectory will likely be distinguishable by the presence of a dust cloud or tail. We do find that a very small fraction of Earth impactors are located near the nominal on-sky position of Apophis despite having received an impulse more than 30 days before the simulated observation, though none more than 180 days. This suggests that single-night observations of Apophis are likely to be very helpful in identifying dangerous scenarios, but may not be sufficient on their own.

During the late observing window, the majority of the Earth-impacting clones are well away from the nominal ephemeris position. A few of these cases fall within 1 arcsecond of nominal on the sky (Figure 9); however, all such scenarios examined in this study occured when the impact that deflected Apophis happened within the 20 days prior

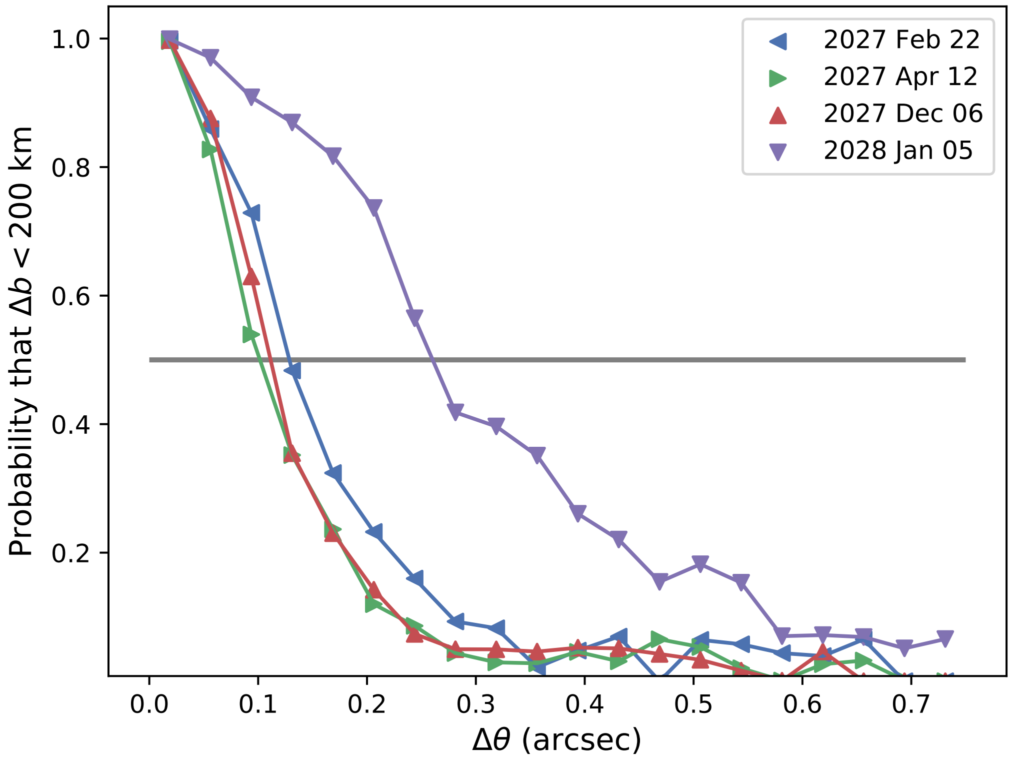

Figure 7. The probability that an observed on-sky displacement $\Delta\theta$ from the nominal Apophis ephemeris is associated with a target plane displacement $\Delta b < 200$ km, for different dates within the early and late observing windows. A high probability indicates Apophis is unlikely to have moved substantially with respect to important keyholes. A horizontal line is drawn at a probability of 0.5.

to the observation. Given the persistence of the resulting dust cloud, this case should be identifiable reliably from telescopic measurements.

3.3.3. *Other observables*

Since our goal is the rapid assessment of whether or not Apophis might have undergone a perturbation of concern, we consider briefly whether other simple observables could be used to assess Apophis' post-2027 orbit.

The apparent magnitude of an asteroid can be used as a proxy for its distance, if its absolute magnitude is well known. But if a small asteroid does deflect Apophis, its surface properties and reflectivity could be changed by the impact. There could also be residual dust in orbit of Apophis that makes using its apparent magnitude to make deductions about its trajectory difficult. But a distinct brightening of Apophis observed in 2027 could be the result of an impact, and would indicate that further investigation was warranted.

Additional orbital information could be provided by parallax measurements of distance. Though not traditionally used in orbit determination, parallax-based techniques are becoming more prevalent (Heinze & Metchev 2015; Gural et al. 2018; Zhai et al. 2022). Two stations on opposite sides of the Earth observing Apophis when it emerges from the daytime sky in 2027 will see it at a distance of approximately 1 au, and so would measure a parallax p

$$p = \frac{2R_E}{1 \text{ au}} \approx 18 \text{ arcseconds}$$

where R_E is Earth's radius. The parallax could be used to determine Apophis' distance if it could be measured precisely enough. A longer baseline would assist in parallax measurements. The Earth's orbital motion about the Sun provides roughly 2.5 million km of baseline per 24 hour period, but parallax measurements would be complicated by

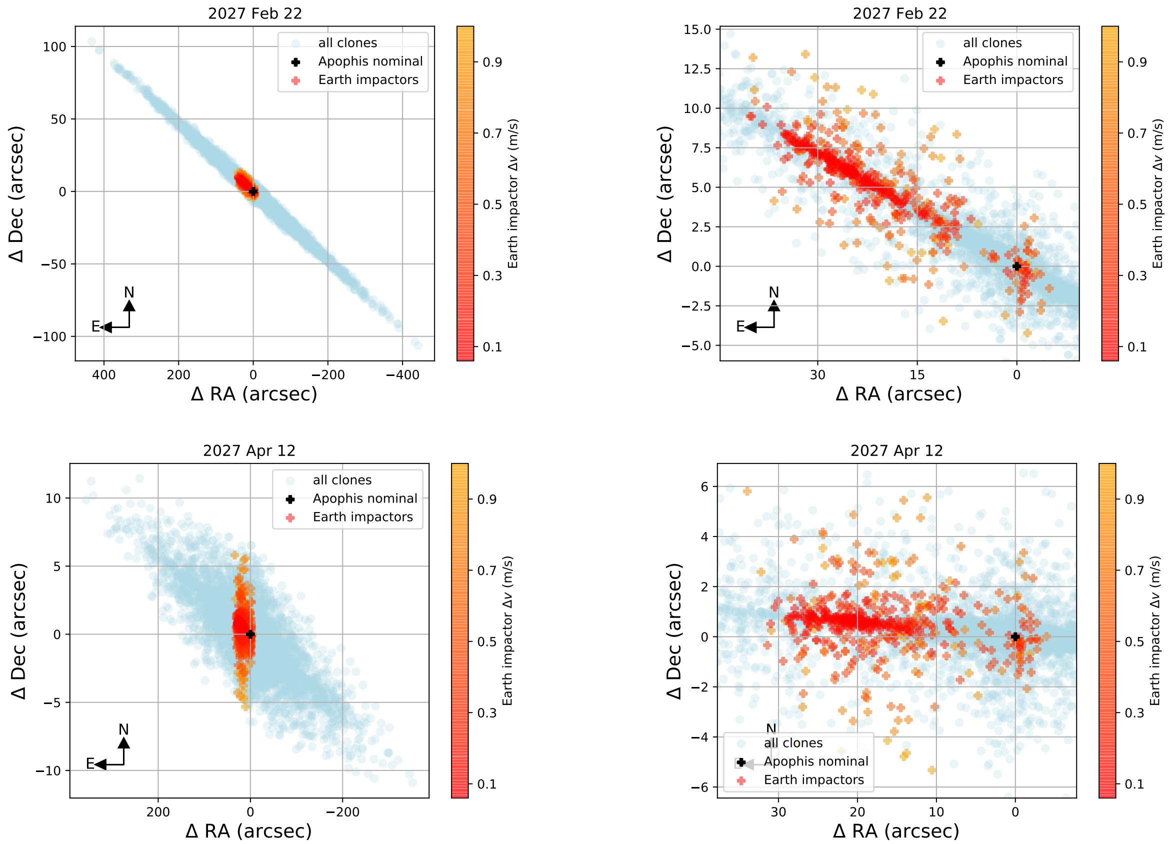

Figure 8. The on-sky position of all clones for the beginning (top) and end (bottom) of the early observing window, at two different magnifications. Clones that would impact Earth are indicated by crosses colored by the Δv received.

the motion of the asteroid during that period of time. A space telescope such as the James Webb Space Telescope (JWST) located at the Earth-Sun L2 point would provide baseline of order 1 million km, much longer than available to Earth observers alone. JWST is not designed to point within 85 degrees of the Sun (Nella et al. 2004), but other current or future space telescopes may be able to contribute to a determination of Apophis' distance via parallax, in the unlikely event that circumstances warrant it.

4. CONCLUSIONS

The odds of a small unseen asteroid colliding with Apophis in such a way as to create a dangerous outcome is exceptionally small. Here we compute the odds of an impact deflecting it significantly relative to known keyholes as less than 1 in a million, and of it being deflected by an amount comparable to its 2029 miss distance as less than 1 in a billion. These probabilities represent only a very low risk, and are below the usual thresholds considered by asteroid impact warning systems (typically 10^{-6}, Milani et al. (2005); Roa et al. (2021)).

Apophis will remain largely unobservable until 2027. When it does return to visibility, a displacement of this asteroid's on-sky position by more than a few tenths of an arcsecond from nominal during 2027 could indicate that it has been perturbed so as to have changed its target b-plane intersection substantially relative to important keyholes. Such a displacement —if observed— would not necessarily indicate that it had been deflected onto a dangerous trajectory, but would indicate that follow-up and analysis should be initiated.

For the particularly unlikely case that Apophis gets deflected onto an Earth-impacting trajectory in 2029, the asteroid's on-sky position in 2027 will differ from the expected ephemeris by tens of arcseconds in almost all cases. It is however possible that Apophis will appear near its nominal on-sky position in 2027 despite being on an Earth-impacting trajectory. These cases correspond to the deflection occuring just before the observations are taken: Apophis will not yet have had time to differ much from its nominal trajectory. Such deflections are found to occur usually within 30 days and no more than 180 days prior for the cases examined here. Such a scenario should be distinguishable by a

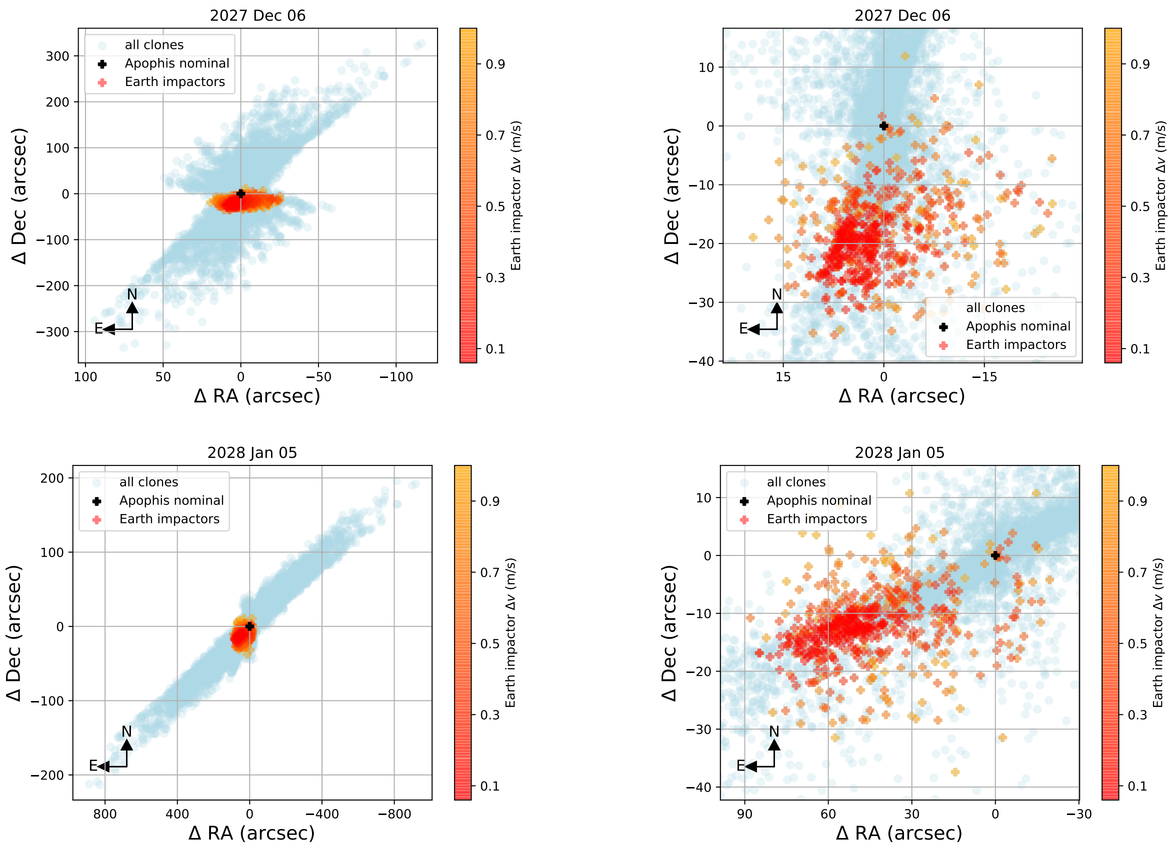

Figure 9. The on-sky position of all clones for the beginning of (top) and 30 days into (bottom) the late observing window, at two different magnifications. Clones that would impact Earth are indicated by crosses colored by Δv.

residual optical brightening of Apophis caused by dust generated from the hypervelocity impact that deflected it. We conclude that the deflection of Apophis by a small asteroid onto a collision course with Earth in 2029 — in addition to being extremely unlikely — will most likely be quickly eliminated as a possibility by simple telescopic observations when Apophis returns to visibility in 2027.

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