

A survey of orbits of co-orbitals of Mars

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Abstract

Many asteroids with a semimajor axis close to that of Mars have been discovered in the last several years. Potentially some of these could be in 1:1 resonance with Mars, much as are the classic Trojan asteroids with Jupiter, and its lesser-known horseshoe companions with Earth. In the 1990s, two Trojan companions of Mars, 5261 Eureka and 1998 VF₃₁, were discovered, librating about the L₅ Lagrange point, 60° behind Mars in its orbit. Although several other potential Mars Trojans have been identified, our orbital calculations show only one other known asteroid, 1999 UJ₇, to be a Trojan, associated with the L₄ Lagrange point, 60° ahead of Mars in its orbit. We further find that asteroid 36017 (1999 ND₄₃) is a horseshoe libration, alternating with periods of Trojan motion. This asteroid makes repeated close approaches to Earth and has a chaotic orbit whose behavior can be confidently predicted for less than 3000 years. We identify two objects, 2001 HW₁₅ and 2000 TG₂, within the resonant region capable of undergoing what we designate “circulation transition”, in which objects can pass between circulation outside the orbit of Mars and circulation inside it, or vice versa. The eccentricity of the orbit of Mars appears to play an important role in circulation transition and in horseshoe motion. Based on the orbits and on spectroscopic data, the Trojan asteroids of Mars may be primordial bodies, while some co-orbital bodies may be in a temporary state of motion.

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1. Introduction

The classic theory of three-body motion describes the possibility of stable companions moving in the same orbit as a planet (parent body) about the Sun (see, for example, Murray and Dermott, 1999). Such motion had from 1906 until 1991 been known to take only the form of librations around the triangular Lagrange points of Jupiter, spaced 60° from the planet along its orbit, and the asteroids involved were called Trojans. In 1991, asteroid 5261 Eureka was discovered to have a similar motion with respect to Mars (Innanen, 1991), and by

extension referred to as a Mars Trojan. A second Mars Trojan, 1998 VF₃₁, was subsequently recognized (Tabachnik and Evans, 1999). More recently a Trojan asteroid of Neptune has also been found (Brasser et al., 2004a). More complex behavior is possible, in the form of horseshoe orbits along the parent orbit, known until now to have heliocentric occurrence only for Earth (Connors et al., 2002) and Venus (Brasser et al., 2004b). Earth's companions have likely been detected only due to the horseshoe orbits having portions very near Earth, allowing them to become relatively bright and favoring discovery. Complex resonant interactions involving high eccentricity, high-inclination objects, such as that of 3753 Cruithne (Wiegert et al., 1997) are also possible. It is now also known to be possible for the companion to remain near the parent planet over a relatively long

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Table 1
Osculating orbital elements on MJD 53200 (cited from AstDys (2004))

Object and class	a (AU)	e	i (deg)	Asc node (deg)	Arg Peri (deg)	M (deg)
5261 Eureka L ₅ Trojan	1.52347	0.064834	20.28	245.108	95.361	104.086
1998 VF ₃₁ L ₅ Trojan	1.52417	0.100449	31.295	221.36	310.527	261.524
1999 UJ ₇ L ₄ Trojan	1.52443	0.039258	16.751	347.431	48.315	190.663
36017 HS	1.52268	0.314047	5.556	332.256	51.848	190.537
2001 HW ₁₅ CT	1.52531	0.252965	18.99	56.575	194.709	235.097
2000 TG ₂ CT	1.52165	0.245329	12.0	206.92	200.202	338.201

Classes are Trojan, Horseshoe libration (HS), and Circulation Transition object (CT).

period of time in the quasi-satellite state (Mikkola and Innanen, 1997; Connors et al., 2004b). Co-orbital interaction may dominate a small third body's motion, according to expansions under Hill's theory (Namouni, 1999), if the guiding center (long-term average position) of that body lies in an annulus within a distance εa (the Hill sphere radius) of the semimajor axis of the secondary (planet). Here a denotes the semimajor axis, and for an essentially massless asteroid, $\varepsilon = (m/3M)^{1/3}$, m and M being the masses of planet and Sun, respectively. For the Earth–Sun system $\varepsilon = 0.01$, accurate to five decimal places, for Jupiter $\varepsilon = 0.06827$, and for Mars $\varepsilon = 0.004756$. Mars has a very small co-orbital zone due to its small mass. However, co-orbital behavior could be possible for asteroids having $1.51645 < a < 1.53095$ in units of AU. We note further that a for Mars changes over time, but by considerably less than the width of the co-orbital region for the timescales studied here.

In order to investigate the properties of asteroids possibly in 1:1 resonance with Mars, we have used several sources of orbital information and several different integrators and compared the results among them. In this descriptive paper, we have used dynamical clones as necessary, as has been done in some other cases (e.g. Connors et al., 2002; Brassler et al., 2004a). Despite the orbits of the objects of interest being well determined at the present time, chaotic effects limit the time period over which the motion can be well described. In all cases studied, an initial investigation over a 600 year period was done using the Horizons online system at JPL (Giorgini et al., 1996). We investigated 38 asteroids, comprising all currently known (Minor Planet Center, 2004) to satisfy the co-orbital condition specified above in terms of limits on a , and some others slightly outside those limits. In many cases, it was clear that the objects were only circulating and these cases were not followed up. For objects of interest, a longer run was done using the Mercury integrator (Chambers, 1999). Further calculations on the objects included here were also done with the Wisdom–Holman algorithm (Wisdom and Holman, 1991) with a time step of several days, and in some

cases with another symplectic algorithm (Mikkola and Palmer, 2001). In all cases, substantial agreement between the results of the various codes was seen, to the point that the results of any code would be nearly indistinguishable for near-present times on the plots shown here. Similar considerations to those of Connors et al. (2002) lead us to consider the Yarkovsky effect unimportant for this study.

The orbital investigations, coupled with recent physical observations, allow some insight into the origin of the co-orbital bodies. We have found three asteroids classified as Trojans; one currently performing horseshoe libration; and two capable of making transitions between circulation external to and internal to Mars' orbit (which we call 'circulation transition'). The osculating parameters and our classification of the behavior of these six objects are shown in Table 1. We now describe the results according to the type of motion found.

2. Trojan asteroids

In idealized circumstances, motion exactly at a triangular Lagrange point is possible, but in practice a more complex motion about these points takes place. In the frame that rotates at the planet's rate of revolution about the Sun, the two most important aspects of the motion are epicyclic motion with period close to that of the parent body, and libration, or motion of the center of the epicycle, with a longer period (see, for example, Murray and Dermott, 1999). Although, in extreme cases, the librational motion can approach half the length of the orbit, such Trojan asteroids can be associated at all times with one or the other triangular Lagrange point. That is to say, they may move along the parent body's orbit but remain on only one side of the line from the Sun to that body.

The first Mars Trojan was discovered by Levy and Holt, and recognized as a likely Trojan by Bowell, in 1990 (Innanen et al., 1991; Innanen, 1991). Now known as 5261 Eureka, it was noted due to its semimajor axis being close to that of Mars and being very near the L₅

Lagrange point. The object 1998 VF₃₁ was recognized as an L₅ Mars Trojan, despite not being extremely near the L₅ point, and several possible other candidates were also found, in 1998 (Tabachnik and Evans, 1999). There have been claims that there were up to six Mars Trojans and that they were all located near the L₅ point (Marzari et al., 2002), but our integrations show that only Eureka and 1998 VF₃₁ are currently L₅ Mars Trojans, as recently noted by Scholl et al. (2004). Numerical experiments allowed Tabachnik and Evans (1999) to confirm the finding of Mikkola and Innanen (1994) that Mars Trojans are stable only when in notably inclined orbits. As may be seen in Table 1, the two L₅ objects have semimajor axes very close to that of Mars (1.5237 AU in 2004), and relatively low-eccentricity orbits. The inclination of Eureka is 20°, while that of 1998 VF₃₁ is over 31°. Both of the L₅ Trojans are in the long-term stability zones found by Tabachnik and Evans (1999), leading them to surmise that these could be primordial objects in the sense that the orbital configuration may date to the time of the early Solar System. Rivkin et al. (2003) investigated spectroscopically, and found that these two asteroids are of similar composition, being stony asteroids of unusual Sr or A class. In contrast, the Mars L₄ Trojan candidate 1999 UJ₇ was found to be of X or possibly T class, which, while of unclear mineralogy, is distinct from that of that of Eureka or 1998 VF₃₁.

We have investigated the orbit of 1999 UJ₇ and find that it is the only currently known Mars Trojan associated with the L₄ Lagrange point of Mars, preceding it in its orbit. Due to being in a different position relative to Mars, and with a different spectrum, 1999 UJ₇ may not have a direct connection to the other Mars Trojans. We now proceed to general discussion of the properties of the three known Mars Trojans.

To illustrate the orbital characteristics of Mars Trojans, we show in Fig. 1 the orbits of Eureka, 1998 VF₃₁, and 1999 UJ₇, in the frame co-rotating with Mars. Since Mars has a rather eccentric orbit ($e > 0.093$), some aspects of the motion in the co-rotating frame are a result of the relative slowing and speeding up of the bodies due to Kepler's Second Law. It would be possible to use a frame rotating at the average rate of Mars, but we have preferred to illustrate the motion as seen from the planet itself. In the cases of the other planets with known co-orbital companions (Venus, Earth, Jupiter, and Neptune), the eccentricity is low enough that the distinction would be small. During one Mars year, the apparent motion of a Trojan asteroid is a distorted, nearly closed loop. Over longer term, the centers of these loops drift back and forth along Mars' orbit, making a libration.

The motion is shown in Fig. 1 over one half libration period, during which the asteroids move along Mars' orbit, and in the cases of 1998 VF₃₁ and 1999 UJ₇,

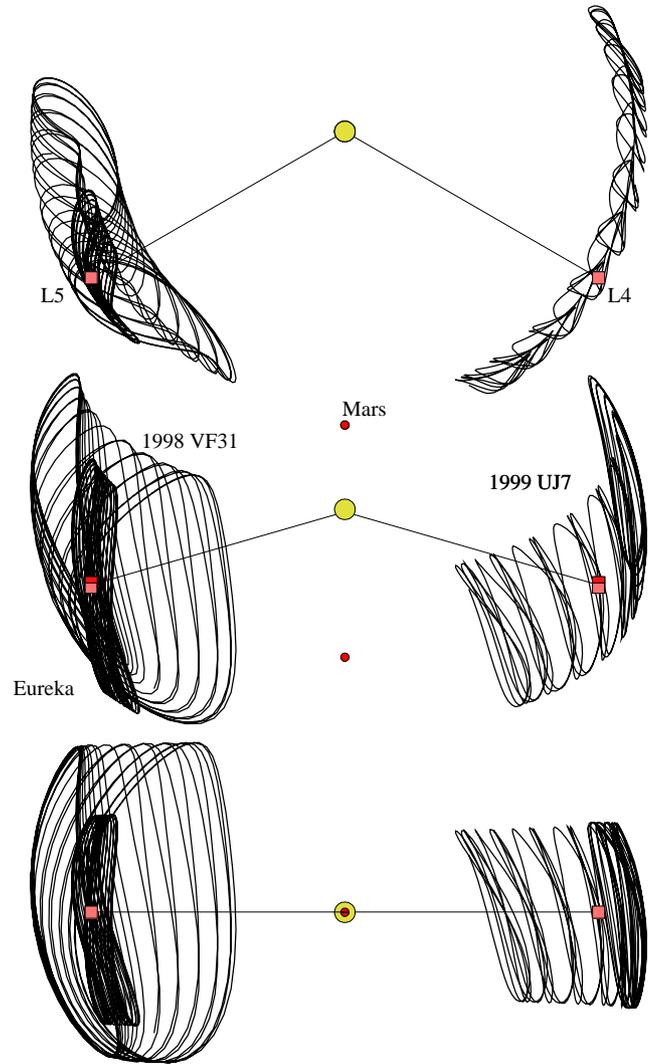


Fig. 1. Motion of Mars Trojan asteroids in a frame co-rotating with Mars. Views are shown to illustrate the three-dimensional nature of the orbits relative to Mars over one-half libration period for each object. These views are, from top to bottom: from the north pole of Mars' orbital plane; at a 30° angle above the radius vector from Mars to the Sun; and along this radius vector. The L₄ or leading Lagrange point is shown by a box at right, the L₅ or trailing Lagrange point by a box at left. Mars is shown by a small filled circle, and the Sun by a larger filled circle, neither to scale with the overall plot.

relatively far from their respective Lagrange points. The apparent thickness of the motion as seen from the side is due to the relatively high inclinations which appear to be required in order to show stable Trojan motion. For 1998 VF₃₁, the approximate libration period is 1400 years, while for 1999 UJ₇ it is 1500 years. 1998 VF₃₁ has a rather high inclination of over 31°, and thus its motion features large up and down excursions: those of 1999 UJ₇ are more limited as its inclination is only approximately 17°. The libration amplitudes of each of these asteroids about their respective Lagrange points are large but similar: 70° in the case of 1998 VF₃₁ and

80° in the case of 1999 UJ₇. We note that 5261 Eureka has a much smaller librational range than the other two Mars Trojan asteroids. It remains rather near the L₅ Lagrange point, with libration period of 1250 years. As noted by Tabachnik and Evans (1999), the Lagrange point also moves and the total range of motion is less if corrected for this.

The orbits of the known Mars Trojans are well established due to long observational arcs. They can come close to the Earth, thus becoming relatively bright, and observations can be obtained for long periods of time compared to those available in which to observe typical near-Earth asteroids. Even for 1999 UJ₇, discovered in 1999, predisccovery images allow the arc to be extended back to 1955, so that the orbit is very secure and residuals (errors) very small (AstDys, 2004).

Trojan motion is possible because an L₄ asteroid approaches Mars on the leading side because of its larger semimajor axis, which according to Kepler's Third Law gives it a mean motion slightly slower than that of the planet, which therefore catches up. The attraction of Mars' gravity backwards along the orbit removes energy from the asteroid, causing its semimajor axis to decrease and the mean motion to increase, so that it pulls away from the planet. When it gets further away, the opposite happens. This causes a long-term to and fro motion along Mars' orbit, the libration. The phasing is reversed for an L₅ asteroid, but the basic mechanism is identical. One result of this motion is a cyclical change in semimajor axis a , which librates about the value associated with the planet. We illustrate in Fig. 2 the variation in a for all bodies studied, and the Trojans show periodic sawtooth waves in this plot.

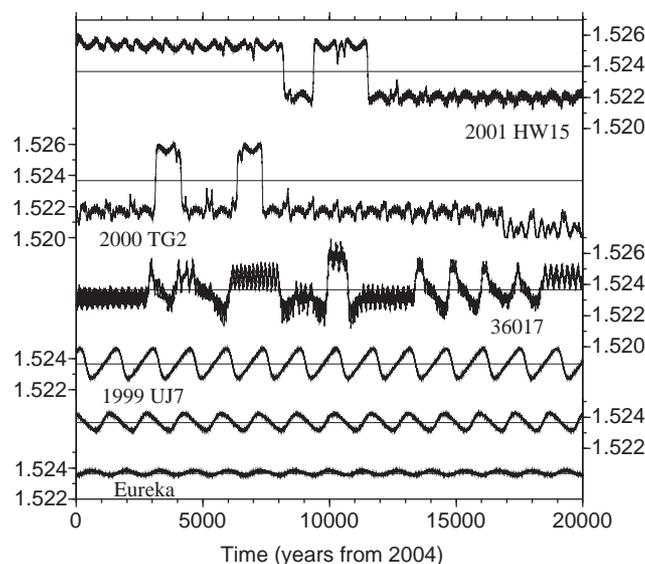


Fig. 2. Evolution of the semimajor axis a with time for Mars co-orbital objects. All objects are plotted at the same scale, with appropriate scale bars for each on the side closest to its label. In each case, the current semimajor axis of Mars is shown by a horizontal line.

The low-amplitude variation of Eureka is almost sinusoidal, and the two other objects, which have a greater range of libration in longitude, also show larger variation in a .

3. Horseshoe orbits

The possibility of horseshoe orbits had been proposed by Brown (1911), shortly after the discovery of Jupiter's Trojans. Such orbits can be stable over finite periods of time for several planets, including those of the inner solar system (Tabachnik and Evans, 2000). The first co-orbital horseshoe orbits known were in the Saturnian satellite system (Dermott and Murray, 1981). The first identified heliocentric horseshoe orbit, that of 3753 Cruithne relative to Earth, (Wiegert et al., 1997) is not co-orbital in the sense that word had for the Trojan asteroids, due to large inclination and eccentricity. Asteroid 2002 AA₂₉ was noted as having a very Earth-like orbit and this gives rise to what may be regarded as more typical horseshoe motion (Connors et al., 2002). Other horseshoe asteroids of Earth have subsequently been recognized (Connors et al., 2004a; Connors and Innanen, 2004). Mars at present has only one horseshoe asteroid, according to our integrations of possible co-orbital candidates. The horseshoe object is asteroid 36017, originally designated 1999 ND₄₃. This asteroid has been investigated spectroscopically and found to be of Bus type S1 (Binzel et al., 2001). This makes it a relatively common class of stony asteroid (Bus and Binzel, 2002). As noted above, Mars Trojans are also stony, with the L₅ objects being of an unusual type.

The motion of this object is more complex than that of any other known horseshoe liblator. This is mainly due to the fact that its relatively large eccentricity of over 0.31 allows close approaches to Earth. An initial appreciation of the complexity may be had by examining the variation in semimajor axis a shown in Fig. 2. In contrast to the regularity of a variation of the Trojan asteroids, that of 36017 is irregular on both short and long timescales. Even when compared to that of the circulation transition objects (to be discussed below), the semimajor axis variation is complex. High frequency variations in a cause an apparent large thickness of the trace of this object at most times shown in Fig. 2. This is due to relatively frequent close encounters with Earth, which cause rapid, if small, changes in semimajor axis. This “fuzz” is illustrated more clearly with an expanded timescale in Fig. 3, which shows nearly periodic changes in a with about 50 years between pairs of downward and upward transitions, each pair being separated by nearly 20 years. The range to Earth is also plotted there, as an aid to seeing the importance of close approaches to Earth as well as their current remarkable periodicity. As may be verified in Fig. 2, near the present time a is

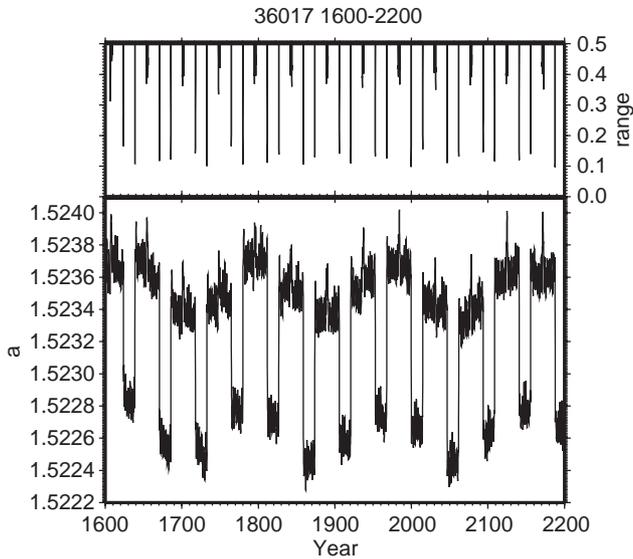


Fig. 3. Bottom panel: evolution of the semimajor axis a of Mars horseshoe orbiter 36017 over the 600 year period 1600–2200. The abrupt changes of semimajor axis on a decadal time scale are due to close approaches to Earth. Top panel: range from Earth. Each significant downward transition of a is associated with the first of a pair of close (ca. 0.1 AU) approaches to Earth, with an upward transition on the next similar approach. Smaller changes in a are associated with approaches to ca. 0.4 AU, which occur between each pair of close approaches.

almost always slightly smaller than that of Mars. The overall result is very slow counterclockwise motion in the corotating frame, with arrival near Mars in about 2500 years, whereupon an interaction takes place to move the orbit to larger a . Our studies of up to 300 clones indicate that the motion cannot be reliably studied past this time. The most likely outcome is that the object will begin to retrace its path, but not make it past 180° , becoming an L_5 Trojan. Subsequent motion is not discussed in detail here but is similarly complex in most cloned situations. As Fig. 4 shows, small variations in a produce quite distinct future behavior, so that motion after 2500 years is not determinable due to chaos. In general, the later motion typically features changes between horseshoe motion and Trojan states, and apparently always with interaction with Earth.

4. Circulation transition

Asteroids 2000 TG₂ and 2001 HW₁₅ are presently circulating at smaller and larger semimajor axis a than is Mars, respectively, as may be seen in Fig. 2. In roughly 3500 years, 2000 TG₂ will undergo a transition to larger a , while in roughly 8000 years, 2001 HW₁₅ will undergo a transition to smaller a , each reversing its average distance with respect to that of Mars. Interestingly, the basic behavior while circulating, with superposed high

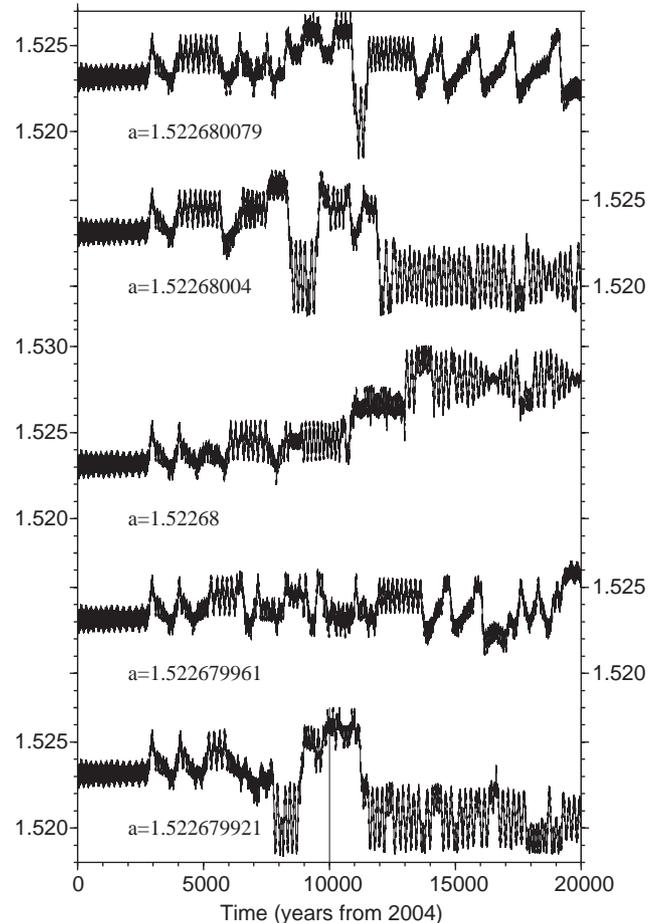


Fig. 4. Sensitivity of the orbit of asteroid 36017 to small changes in present conditions. The semimajor axis a has been varied about its nominal value by amounts comparable to its present uncertainty of approximately 8×10^{-8} AU. The central trace shows the nominal motion; those immediately above and below traces the motion with plus or minus half the uncertainty, respectively; and the top and bottom traces the motion with plus or minus the uncertainty, respectively. After the encounter with Mars in about 2500 years, the motion is different in all of these cases.

and low frequency perturbations, remains the same before and after transitions, and is similar for both objects. In both cases, the period in the opposite semimajor axis state lasts about 1000 years, followed by a transition back to the present state. We note, however, that this regularity is not absolute since 2001 HW₁₅ returns to the present state, after its initial flip-flop, for only about 2000 years, and then makes a long-lasting transition to circulation at smaller semimajor axis than Mars. We illustrate in Fig. 5 the semimajor axes and distance from Mars of 2001 HW₁₅ and 2000 TG₂ when near transitions, to show that the transitions take place when the objects are near Mars. We infer that the transitions are possible since the large relative eccentricity allows a close approach to Mars, but that this depends sensitively on the relative orbital

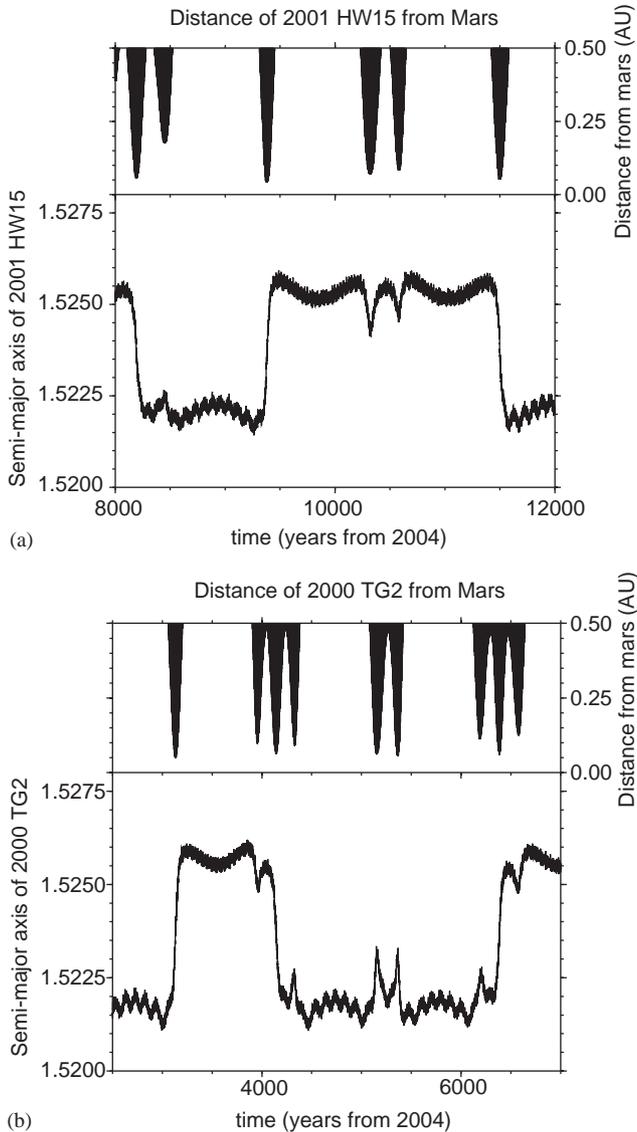


Fig. 5. Circulation transition and distance from Mars. (a) The semimajor axis of 2001 HW₁₅ is shown (below) near times of the three circulation transitions shown in Fig. 2. In the upper panel the corresponding distance to Mars is shown. Periods when the relative orbital geometry allows close approach correspond to changes in the semimajor axis of the asteroid. (b) The semimajor axis (below) of 2000 TG₂ is shown for times covering the first three of the four circulation transitions shown in Fig. 2. As in Fig. 5a, close approaches (distance shown above) correspond to times of change of a .

parameters as they evolve over time. The behavior of the circulation transition objects is not as chaotic as that of present horseshoe librator 36017, likely due to the eccentricity being less and not allowing close approaches to Earth. We do note that both objects in this class have relatively high eccentricity, around 0.25, and we suspect that this, and the relatively high eccentricity of Mars, plays a significant role in the new type of behavior we note here. We leave a more detailed study for later and suggest that the behavior noted here for the first time

with real objects may correspond to the “passing orbits” shown to be theoretically possible by Namouni (1999).

5. Discussion

The general question of the origin of co-orbital asteroids has been discussed extensively, particularly with regard to Trojan asteroids, since only that class has been known until recently. The Trojans of Jupiter are generally regarded as primordial (Marzari et al., 2002). Long-term orbital integrations can be done for the outer solar system for timescales of order of the age of the solar system, and these confirm orbital stability for the Jupiter Trojans. In addition, the spectral type of the Jupiter Trojan asteroids is generally D, a low-albedo, reddish type possibly associated with carbonaceous chondrites (Barucci et al., 2002). This is consistent with primitive material and formation in the outer solar system, although it is difficult to specify exactly where. A study of diffusion speeds in orbital parameter space for hypothetical Mars Trojans showed that there is a possibility of lifetimes as long as that of the solar system (Scholl et al., 2004), while integrations over shorter timescales for real objects also showed stability (Tabachnik and Evans, 1999; Mikkola and Innanen, 1994). The spectral evidence is less conclusive as to origins although the finding that the two L₅ Mars Trojans are of a similar rare stony type (Rivkin et al., 2003) is not inconsistent with formation in the vicinity of Mars. On the other hand, the difference of spectral type between the L₄ and L₅ Mars Trojans could suggest capture. That the horseshoe object, less likely to be of primordial origin, has a common stony type, suggests the danger of reading too much into spectral types at this stage of investigation, since stony types are common in the inner asteroid belt which could be a reservoir of transient material. Although the timescales we have investigated are too short to allow us to definitively determine this, the chaotic behavior noted for the horseshoe and circulation transition objects suggests that they are not long-lived in these states. They may fit rather into a larger picture in which bodies move in orbital parameter space, occasionally being trapped into resonance (Christou, 2000; Morais and Morbidelli, 2002).

In a similar way, the origin of the satellites of Mars remains a subject of discussion. Their composition was originally thought to be similar to that of carbonaceous chondrites (Burns, 1992) and dissimilar from the apparently stony composition of the Mars Trojans. This composition argues for satellite capture involving material from the outer solar system, while dynamical arguments including the low-inclination, near-circular orbits, argue for in situ formation near Mars (Peale, 1999). More recent work (Rivkin et al., 2002, and references within) suggest that the satellites of Mars may

have heterogeneous composition and not be very similar to carbonaceous chondrites; further they are not similar to at least the L_5 Mars Trojans. If our inference that Mars Trojans are primordial is correct, it implies formation of stony accretion remnants in the vicinity of Mars. At this stage, however, neither the spectral nor dynamical investigations lead to a consistent picture of the origins of small bodies accompanying Mars.

We note that the objects listed in Table 1 fill a limited part of the a range, already very small, which nominally could include Mars co-orbital motion. We have integrated the orbits of over 1500 Jupiter Trojan asteroids, and in addition to finding no horseshoe librators, note that those co-orbitals also are restricted to a small part of the nominal (and large) a range nominally possible for Jupiter co-orbital motion. This suggests care in the use of ranges of a to classify objects: a different criterion than the Hill annulus ϵ should be used, or, preferably, numerical integration should be used to verify the type of motion.

Finally, although small numbers make it hazardous to extrapolate, the presence of only one Mars horseshoe object and three Trojans may be suggestive for the case of Earth. The Mars objects are in a relatively well-sampled zone of the solar system due to its ability to come to opposition. Since Earth at this writing has five horseshoe objects but no known Trojans, the Mars results suggest that the poorly sampled triangular Lagrange points of the Earth, which are difficult to observe, may harbor Earth Trojan asteroids.

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