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THE STABILITY OF QUASI SATELLITES IN THE OUTER SOLAR SYSTEM

Paul Wiegert and Kimmo Innanen
Department of Physics and Astronomy, York University, Toronto, ON M3J 1P3, Canada; and
Canadian Institute for Theoretical Astrophysics, University of Toronto

and

Seppo Mikkola
Tuorla Observatory, University of Turku, Väisäläntie 20, FIN-21500 Piikkiö, Finland
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ABSTRACT

Quasi satellites are bodies in a particular configuration of a 1:1 mean motion resonance, one in which they librate about the longitude of their associated planet. We investigate numerically the stability of such orbits around the giant planets of our solar system. We find that test particles can remain on quasi-satellite orbits around Uranus and Neptune for times up to 10^9 yr in some cases, though only at low inclinations relative to their accompanying planet and over a restricted range of heliocentric eccentricities. These stable areas are well outside the traditional satellite region. Based on these results, we conclude that a primordial population of such objects may still exist in our solar system.
Key words: minor planets, asteroids —
planets and satellites: individual (Jupiter, Saturn, Uranus, Neptune)

1. INTRODUCTION

The outer solar system is coming to be seen as much less empty than it appeared even 10 years ago. Recent detections of objects in the Kuiper belt beyond Neptune have extended our knowledge of the distant fringe of our planetary system considerably, with 1996 TL_{66} at the extreme with an aphelion distance of 135 AU (Luu et al. 1997), almost 3 times that of Pluto. The known population of Centaurs, possible comet precursors that travel between the orbits of the outer planets, has increased to nine since the discovery of the prototype Chiron (Kowal, Liller, & Marsden 1979), and the possibility of a long-lived belt of material between Uranus and Neptune has been raised (Holman 1997). Even the more traditional search for satellites of the outer planets continues to meet with success, as borne out by the recent discovery of two small satellites of Uranus (Gladman et al. 1997).

The term "quasi satellite" (QS), coined by Mikkola & Innanen (1997), refers to an object in a specific configuration of a 1:1 mean motion resonance, one in which the body librates around the longitude of its associated planet. QSs are an extension of the retrograde periodic orbits of the circular restricted three-body problem (Jackson 1913; Szebehely 1967; Hénon 1969; Hénon & Guyot 1970). When viewed in a frame that corotates with the planet, the QS traces a path around that body over the course of one orbital period, much as a satellite would. This path is retrograde and, at low eccentricities, elliptical with a 2:1 major-to-minor axis ratio (Fig. 1). No bodies are currently known to be in such a configuration.

Here we investigate the stability of such orbits around the giant planets. The outer regions of our solar system seem the most likely to contain such objects, given the existence of numerous particles in the so-called Trojan 1:1 mean motion resonance with Jupiter. Thus, though the possibility that quasi satellites might exist in the terrestrial region is equally exciting, we will not consider this possibility here.

The numerical model used is described in the next section; the results are presented in § 3, and our conclusions follow in § 4.

2. THE MODEL

The numerical algorithm used here is the symplectic method due to Wisdom & Holman (1991). Our model includes the four giant planets but not Pluto or the inner planets, the mass of the terrestrial planets having been added to that of the Sun. Though the inner planets may potentially play a role in the removal of Jupiter QSs with high eccentricities ($e \gtrsim 0.7$), such objects are found to become unstable relatively quickly regardless. The possibility that the inner planets could stabilize high-eccentricity Jupiter QSs seems unlikely enough that it is ignored here.

The planets in the model interact gravitationally, but the influence of the test particles, both on the planets and on each other, is neglected. The test particles are laid down on orbits with the same heliocentric Keplerian elements as their planet with the exception of their eccentricity and inclination.

The test particles do not interact with one another, and thus one could conceivably integrate them all simultaneously. However, the simulations were performed separately to allow longer time steps to be used when considering the more distant planets. As a result, each simulation includes only the potential QSs associated with one of the four planets. This procedure reduced both the processor time required for the more distant planets and the accumulation of round-off errors.

The particles are laid out in sets of 50 with nominal heliocentric eccentricities e ranging from 0 to 0.5 for Jupiter through Uranus, and $0 \leq e \leq 0.35$ for Neptune. The upper limits on e were chosen to be near the values at which the particles would begin to cross the orbits of adjacent planets. The exception is Jupiter, QSs of which would need eccentricities in excess of 0.8 to cross both Earth's and Saturn's orbit and $e \gtrsim 0.7$ to cross the orbit of Mars. In this case, a maximum eccentricity of 0.5 was chosen based on preliminary simulations that indicated that Jupiter QSs with such large eccentricities quickly become unstable.

Initially, nine sets of 50 test particles were simulated for each planet, each set at a different relative inclination Δi, for a total of 450 particles per planet or 1800 particles all

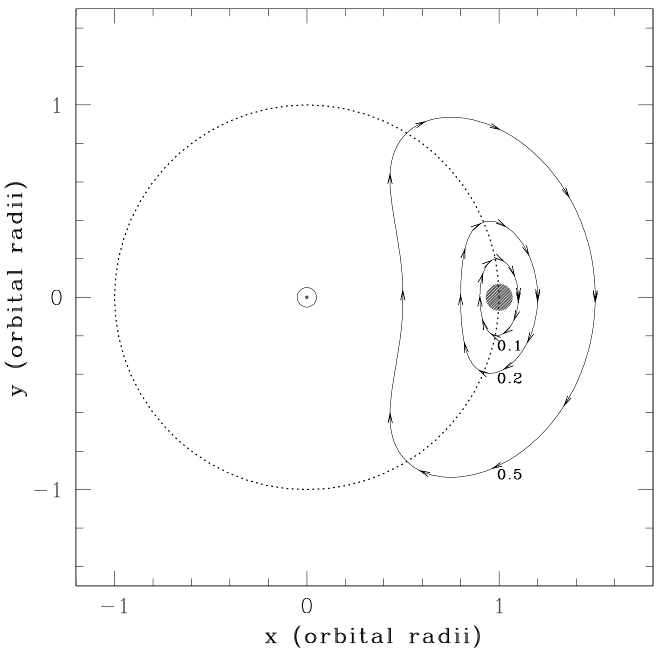

FIG. 1.—Nominal shapes of three QS orbits, seen in a frame that corotates with their planet. These orbits have nominal eccentricities of 0.1, 0.2, and 0.5. The shaded circle is the size of Jupiter's Hill sphere, shown for comparison.

together. The nine sets were given inclinations relative to their planet's orbital plane, with differences Δi[1] of 0°, ±5°, ±10°, ±20°, and ±30°. The ensemble of these 1800 particles will be called the "broad" set. The matching positive and negative inclination increments should produce similar, though not identical, results. These arrangements do not constitute mirror configurations, as the giant planets do not all lie in exactly the same plane. However, we do expect the results to be similar for relative inclinations of the same absolute magnitude, allowing an additional, albeit crude, check on our results.

Based on the simulations described above, further integrations (the "narrow" set) were performed at lower Δi and over a smaller range of eccentricities. Nine sets of 50 particles, 1800 in total, were again simulated at inclinations of −4° to +4°, in 1° increments relative to the associated planet's orbital plane. These simulations provide a more detailed look at the most stable regions of the phase space.

Each simulation in the broad set was performed with two different time steps. The first time step Δt was 0.1, 0.25, 0.5, and 1.0 yr for the QSs of Jupiter through Neptune, respectively, resulting in 120–140 steps per orbital period of the associated planet. The second set was a factor of 5 smaller and was performed as a check on the earlier integrations. The results were qualitatively the same in both cases, so only the results with the larger step size are presented here, unless specifically mentioned otherwise. The simulations in the narrow set were performed with the larger step size only.

We note that the use of larger step sizes for the more distant planets results in a decreased accuracy of the integration of the inner giant planets, which could potentially degrade the accuracy of the simulations of the outer giant planet QSs. However, the time steps chosen allow the necessary accuracy to be maintained. For the most extreme case, when simulating hypothetical QSs of Neptune, a time step of 1.0 or 0.2 yr is used, resulting in approximately 12 or 60 steps per orbit for Jupiter. Given the similarity of the results in these two cases, and the fact that QSs interact most strongly with their associated planet, we conclude that this choice of step size is reasonable.

The difficulties involved in performing such long-term integrations are well known, and we do not claim here to have overcome them. The Lyapunov times of QSs are 10^4 to 10^5 yr for the longest Uranus and Neptune survivors, and thus the strict validity of simulations on longer timescales is questionable. However, we make the conventional argument that the specific details of the evolution are secondary since we are not interested in any specific QS, but only the statistics of the QSs as an ensemble.

Particles passing within the Hill sphere $R_{\rm H} = [M_p/(3\,M_\odot)]^{1/3} a_p$, where M_p and a_p are the mass and semimajor axis of the planet, respectively, are removed from the simulations, as our algorithm is not designed to handle these close encounters to high accuracy. This criterion also prevents confusion (on the basis of their low relative longitudes) of true captured satellites with QSs. Though motivated by practical concerns, this approach is not without some physical justification: one expects particles suffering such close approaches to undergo relatively large changes in their orbital elements, removing themselves from the QS sample. Indeed, our simulations confirm this to some degree: QSs are found to be unstable in the region immediately outside the Hill sphere.

To monitor close approaches, our simulations must reliably detect close encounters between the test particles and the planet. This is done simply by checking at each time step whether any particle is within $R_{\rm H}$ of any planet. This procedure is reliable as long as the distance traveled by a test particle in one time step is much less than $R_{\rm H}$, otherwise the close encounter might be missed. We have verified that the time steps Δt are small enough (even for the largest step size) that the probability of a close encounter being missed by the particle's "stepping over" the Hill sphere without being detected is of order 10^{-2}, and thus we are confident that close encounters between QSs and their planets are detected properly.

Pains are taken to determine which particles in our simulations remain on quasi-satellite orbits and which wander off to nearby regions of phase space. A criterion based solely on the value of the semimajor axis is incomplete because particles may escape into other types of 1:1 mean motion resonances, such as tadpole or horseshoe orbits. Here a particle is deemed to have left the QS state if the longitude difference λ between it and its planet exceeds 120°, since an object would require a heliocentric $e \sim 0.9$ to be on such an elongated QS orbit. However, as this procedure may fail to detect particles orbiting near the L4 and L5 points, we also maintain a record of the particles' minimum and maximum relative longitudes. These values are checked on timescales of about one heliocentric orbital period or less and allow us

[1] Negative inclinations are not properly defined for the standard Keplerian elements, where the inclination is restricted to the range $i \in [0°, 180°]$. In the event of a particle being assigned a negative inclination through the application of the increments mentioned above (i.e., if $i_p + \Delta i < 0$, where the subscript p indicates the planet's ecliptic elements), the following algorithm is used:

$$i = |i_p + \Delta i| \,, \quad \Omega = \Omega_p + 180° \,, \quad \omega = \omega_p + 180° \,.$$

This procedure avoids confusion as to the orbital geometries: all particles are initially on prograde heliocentric orbits.

to determine the QSs' status throughout the simulation. This monitoring also allows a rough examination of the QS-to-Trojan transfer process to be made.

A particle is considered to have left the QS region if it had a close encounter with the planet, if its longitude difference λ exceeded 120° at any point, if its time-averaged semimajor axis differed from its initial semimajor axis by more than 5%, or if λ over the last 10^4 yr of the simulation did not encompass zero. These partially overlapping criteria eliminate all particles that do not remain in 1:1 mean motion resonance with their planet, as well as those on horseshoe or tadpole orbits. These criteria, though perhaps inelegant, are chosen for their efficiency and ease of implementation. Though theoretically we need only monitor the critical angle (i.e., the longitude difference), computer algorithms monitoring solely this parameter can be fooled. For example, the libration of a horseshoe orbit around 180° may be confused with libration around 0°. When the transition between resonant states (QS, Trojan, horseshoe) is considered as well, a simpler method becomes preferable. For example, a single measurement of the semimajor axis of a nonresonant test particle reveals its escape from the mean motion resonance, whereas the critical angle must be monitored for some time to determine that it circulates.

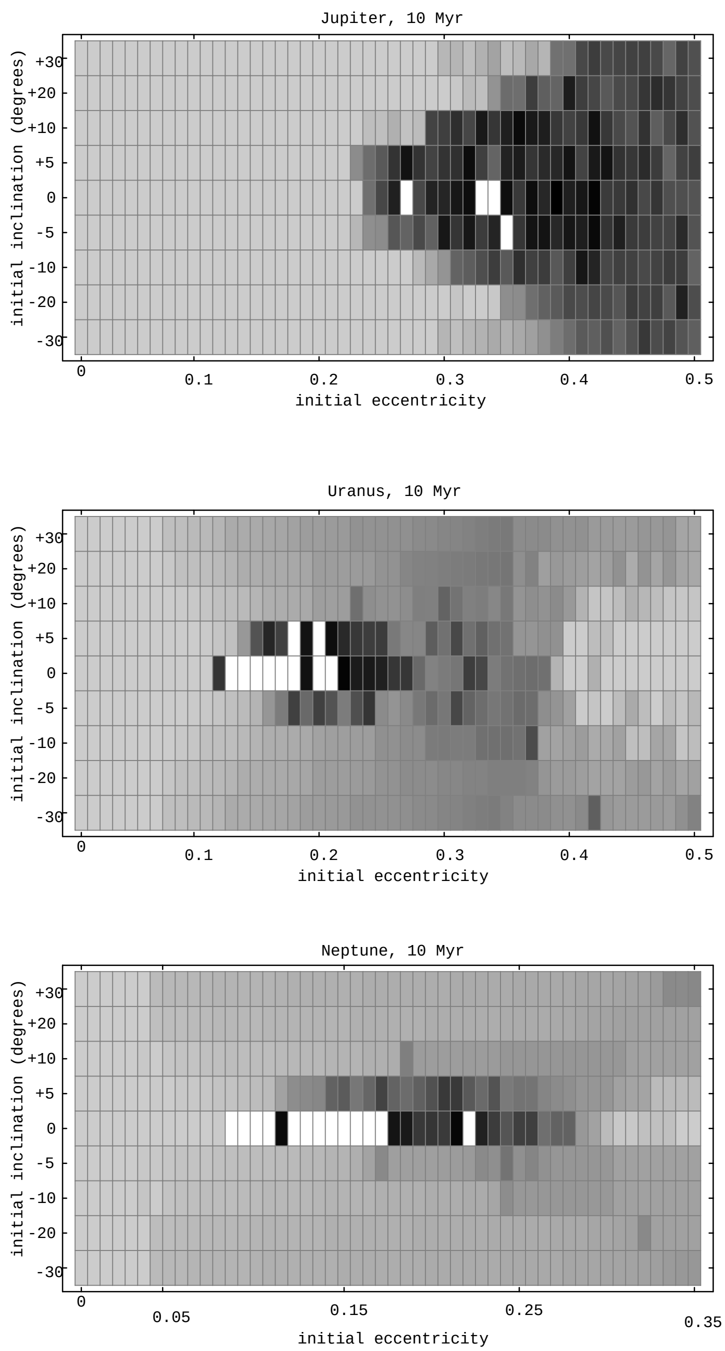

FIG. 2.—End states of the potential QSs of Jupiter, Uranus, and Neptune shown on a grid of initial eccentricity vs. initial relative inclination. Each cell represents a single test particle. Darker cells represent longer lifetimes, with the exception of white cells, which indicate survival for a full 10 Myr.

3. RESULTS

Our results are presented in the form of a grid of rectangular cells. Each cell represents one test particle, and a darker shade indicates a longer particle lifetime. The exceptions are the particles that survive the entire integration length, which are indicated in white for clarity.

The first set of integrations (the "broad" set) is shown in Figure 2. No results are shown for Saturn, as none of its particles survive for even 10^5 yr (the longest lived lasts only 4.3×10^4 yr). Jupiter's QS population is almost all gone by $t = 10^7$ yr (the last particles are lost prior to $t = 50$ Myr), while both Uranus and Neptune maintain a remnant at low inclination. This increased stability at low i is in accord with the analytical results of Mikkola & Innanen (1997).

The surviving particles are also concentrated over a restricted range of eccentricities. We examine this most stable region in more detail with the narrow population, shown in Figure 3. The longest lived QSs of Uranus and Neptune cluster at very low inclinations ($\Delta i \lesssim +2°$) and at eccentricities between 0.1 and 0.15. If particles in this region can survive for the age of the solar system, then a population of QSs may persist in this region. Even if no primordial objects remained or ever existed, the capture of comets from the Kuiper belt or Centaur populations might provide a transient population.

The variations in the eccentricities of the surviving QSs are quite consistent for each planet, with amplitudes of 0.07, 0.07, and 0.02 for the planets Jupiter, Uranus, and Neptune, essentially independent of the QSs' initial or average eccen-

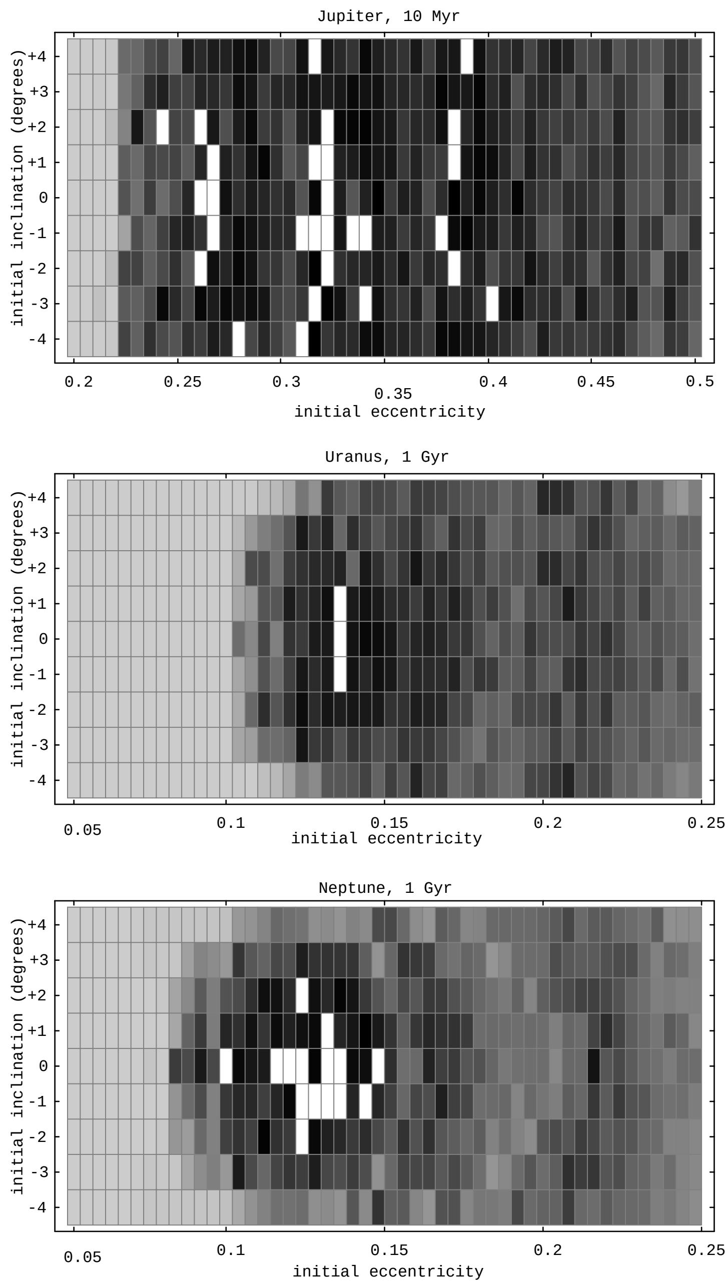

FIG. 3.—Same as Fig. 2, but for the narrow distribution. White cells indicate survival for a full 10 Myr for Jupiter (no particles survive 100 Myr), and 1 Gyr for Uranus and Neptune.

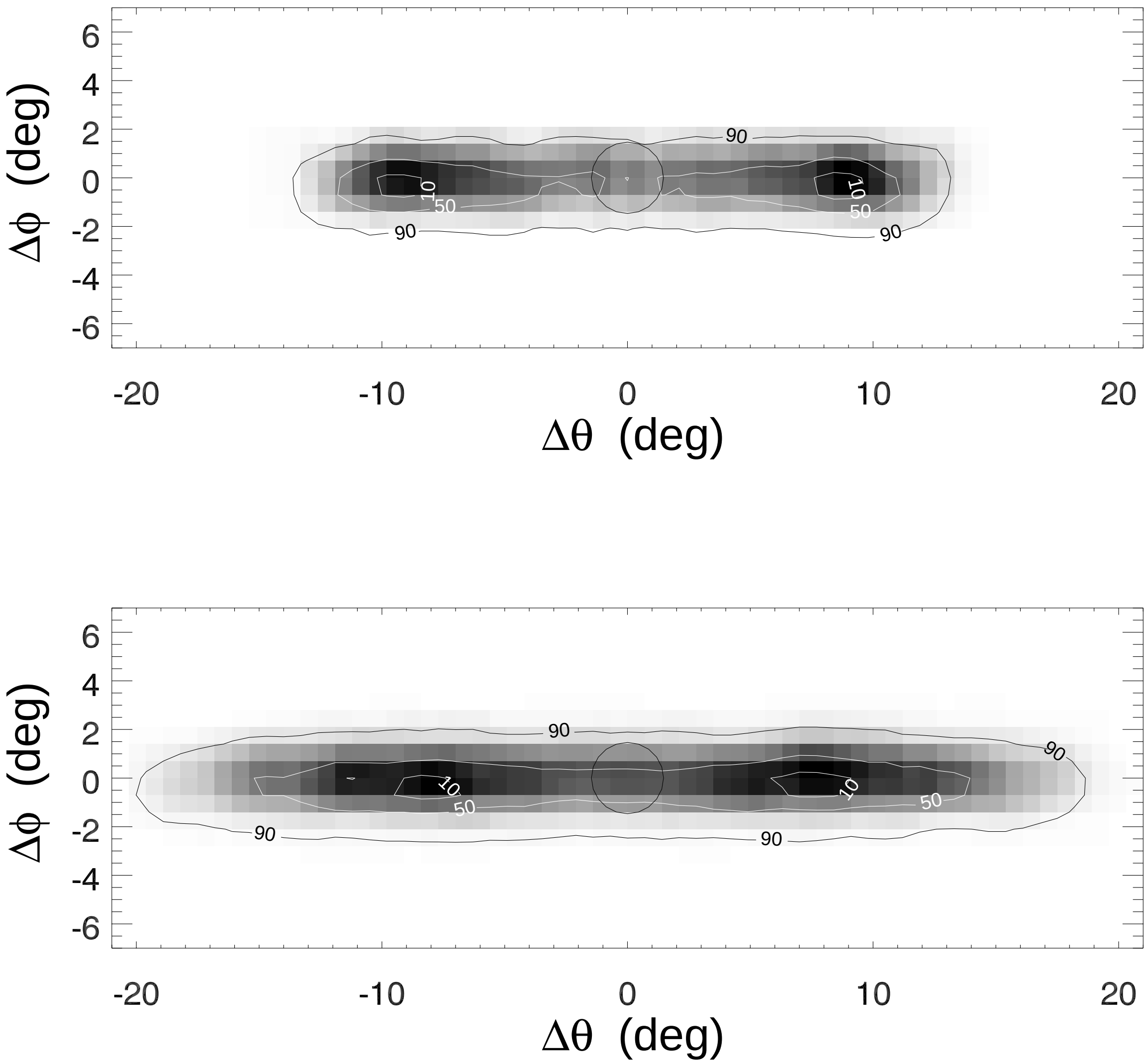

FIG. 4.—Density of the narrow population of QSs surviving 1 Gyr around Uranus (*top*) and Neptune (*bottom*), projected onto the plane of the sky as seen from the Sun. The angle $\Delta\theta$ is measured in the planet's orbital plane, $\Delta\phi$, perpendicularly. The contours are labeled according to the percentage of the data points within them (i.e., the " 50 " contour contains the densest 50% of the synthetic observations) . The circle approximates the planet's Hill sphere.

tricity. The ecliptic inclinations of the surviving QSs remain close to their initial values, typically varying by roughly $\pm 2°$, regardless of the planet or initial inclination in question. This result agrees with the analytical theory of Mikkola & Innanen but constitutes only indirect support of their result (their eq. [14]), as our simulations monitor only the ecliptic and not the relative inclination.

There is no evidence for stable QS orbits near the Hill sphere. The radial half-width $d_{QS} = ae_{QS}$ of a QS orbit as seen in the rotating frame (see Fig. 1) is equal to R_H at an eccentricity $e_{QS} = [M_p/(3\ M_\odot)]^{1/3}$. For the planets Jupiter through Neptune, this corresponds to values of e_{QS} of 0.07, 0.046, 0.024, and 0.026, respectively, at $\Delta i = 0$. We note that particles in the vicinity of the Hill sphere quickly become unstable, though we remind the reader that those crossing the Hill sphere are removed automatically from the simulation. Still, there is no evidence for stable QS orbits with $d_{QS} \gtrsim R_H$; rather, the stable areas are significantly removed from the planet's vicinity, well outside the conventional satellite regions of the planets.

We make a crude evaluation of the QS-to-Trojan transition probability by recording each particle's minimum and maximum longitude difference λ, these values being reset at an arbitrarily chosen interval, in this case 10^4 yr. A particle is considered to have been transferred to a Trojan orbit if its final recorded maximum/minimum λ (whether it survived the whole integration span or was otherwise terminated) encompasses either of the triangular Lagrange points, but not the planet ($\lambda = 0$). This is a rough-and-ready criterion, but it allows us to make a preliminary analysis of the transfer process. In fact, the transfer of a particle from QS to Trojan is rare; only 12 such events are recorded. However, their validity is dubious, each particle involved having been terminated almost immediately after the lapse of a 10^4 yr interval. Under these conditions, the recently reset maximum and minimum values of λ are misleading. It appears that the transfer of particles from QS to Trojan orbits is of low probability.

Figure 4 shows the density of the narrow population of QSs remaining after 1 Gyr, projected onto the plane of the sky as seen from the Sun, for both Uranus (three survivors) and Neptune (15 survivors). Detecting such a population

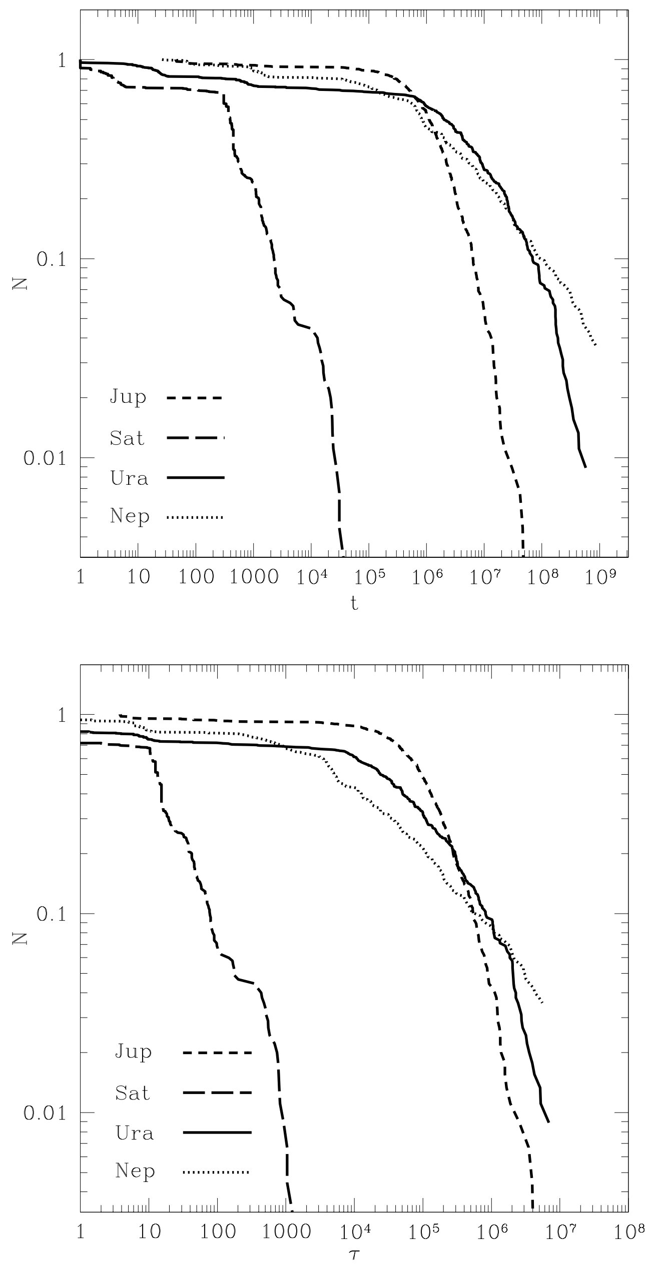

FIG. 5.—Logarithm of the fraction of quasi satellites remaining vs. that of the elapsed time t in years or τ in orbital periods for the narrow population.

would be observationally challenging because of the large angular extent of the cloud. In each case, only $\sim 10\%$ of the density is within the Hill sphere, which has an angular diameter of roughly 3° (for comparison, the angular diameters of Uranus and Neptune themselves are only 4″ and 2″, respectively). The highest number density of QS positions is offset from the two outermost planets by $\sim 8°$, and the overall cloud extends to 15°–20° to either side, though only about 2° perpendicular to the planet's orbital plane.

We can use these numbers to estimate the likelihood of a serendipitous detection of a QS in an image of the vicinity of one of these planets. The projected density at any given instant in the area of the Hill sphere is roughly $3 \times 10^{-3} N$ deg^{-2}, where N is the number of QSs, and there would be $8 \times 10^{-5} N$ objects per $10' \times 10'$ field. Inside the 10% contours, the average surface density is nearly twice as high, roughly $6 \times 10^{-3} N$ deg^{-2}. Thus, a survey taking a handful (~ 10) of small fields near the highest surface density points could expect to be successful only if the planets support QS populations of order 10^3 objects.

In Figure 5, we plot the fraction N of QSs remaining versus the time on a log-log plot for the narrow population. We have tried alternatives, but the relation seems to be better fitted by a power law at late times than by an exponential or logarithmic decay. Figure 5 shows N versus the number of orbital periods of the accompanying planet. This indicates that Jupiter, Uranus, and Neptune quasi satellites are likely subject to qualitatively similar dynamical processes. However, Saturn loses its QS population orders of magnitude faster, possibly indicative of an additional mechanism such as a resonance acting in this case. It is not clear a priori why Saturn should lose its quasi satellites more quickly. Studies of outer planet Trojan asteroids have shown them all to be stable up to the same 10^7 yr timescale examined (Innanen & Mikkola 1989; Mikkola & Innanen 1992; Holman & Wisdom 1993). On the other hand, the lack of detections of Saturn Trojans hints that instability in that planet's coorbital population occurs on shorter timescales than in that of Jupiter. At this time, the differences between these two planets' Trojan and QS populations are not understood.

Fitting a simple least-squares line to the portion of the curve beyond 10^7 yr yields a fraction remaining N of order $0.1[t/(10^8 \text{ yr})]^{-1.45}$ for Uranus and $0.1[t/(10^8 \text{ yr})]^{-0.47}$ for Neptune. If this relation continues to hold, we might expect of order 0.05% or 1%, respectively, of the low-inclination quasi satellites of Uranus and Neptune to remain. Though not conclusive, this analysis suggests that a fraction of a population of QSs with very low inclinations to the outer giant planets could persist for the age of the solar system.

We can use known results about the Jupiter-family comets (JFCs) to put an upper limit on the population of Neptune QSs. Based on their simulations and the results of Fernández, Rickman, & Kamel (1992), Levison & Duncan (1997) calculated that there are ~ 100 active JFCs with perihelia inside 2.5 AU and with a total magnitude less than 9. They also found that roughly 30% of Neptune-crossing objects become Jupiter-family comets, and that the lifetime of these comets is likely 12,000 yr. This means Neptune currently would have to be losing approximately 0.03 QSs per year in order to supply the JFCs. Differentiating our empirical function for N deduced in the previous paragraph, an upper limit of 3×10^8 for the current population of Neptune QSs can be deduced. Clearly, the current absence of QSs discoveries does not bode well for the hypothesis that they provide a significant fraction of the Jupiter-family comets, but because of the selection effects inherent in surveys of the outer solar system, real constraints on the numbers of extant QSs are difficult to construct.

4. CONCLUSIONS

Quasi satellites can persist for times of 10^9 yr and probably longer around the planets Uranus and Neptune, but they become unstable on much shorter timescales for both Jupiter (10^7 yr) and Saturn ($< 10^5$ yr). They are most stable when in the plane of their associated planet, and over a restricted range of eccentricities (0.1–0.15 for Uranus and Neptune).

Because of this stability, we predict that Uranus and Neptune have such populations, though only fairly weak constraints can be placed on their numbers. Jupiter and Saturn are very unlikely to have significant numbers of primordial quasi satellites, because of the short lifetime of such objects; however, the possibility remains that they have transient populations of such objects.

We also note that, should Uranus and Neptune possess primordial clouds of quasi satellites, those being lost currently would have very low inclinations. Any escaped QS would be on a Uranus- or Neptune-crossing orbit and, thus, has the same probability of being scattered farther down into the planetary system as a similar object originating in the Kuiper belt. This raises the possibility that these populations, should they exist, may provide some of the short-period comets, which are known to have generally low inclinations. In the event that Neptune QSs do provide a significant fraction of the Jupiter-family comets, a large current population ($\sim 10^8$) of such objects would be required as a consequence of the present slow loss rate.

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REFERENCES

Fernández, J., Rickman, H., & Kamel, L. 1992, in Periodic Comets, ed. J. Fernández & H. Rickman (Montevideo: Univ. Repub. Dept. Publ.), 143
Gladman, B. J., Nicholson, P. D., Burns, J. A., Kavelaars, J. J., Offutt, W., Tholen, D. J., Herrick, C., & Whiteley, R. J. 1997, IAU Circ. 6764
Hénon, M. 1969, A&A, 1, 223
Hénon, M., & Guyot, M. 1970, in Periodic Orbits, Stability and Resonances, ed. G. E. O. Giacaglia (Dordrecht: Reidel), 349
Holman, M. J. 1997, Nature, 387, 785
Holman, M. J., & Wisdom, J. 1993, AJ, 105, 1987
Innanen, K. A., & Mikkola, S. 1989, AJ, 97, 900
Jackson, J. 1913, MNRAS, 74, 62
Kowal, C. T., Liller, W., & Marsden, B. 1979, in IAU Symp. 81, Dynamics of the Solar System, ed. R. L. Duncombe (Dordrecht: Reidel), 245
Levison, H. F., & Duncan, M. J. 1997, Icarus, 127, 13
Luu, J., Marsden, B. G., Jewitt, D., Trujillo, C. A., Hergenrother, C. W., Chen, J., & Offutt, W. B. 1997, Nature, 387, 573
Mikkola, S., & Innanen, K. 1992, AJ, 104, 1641
———. 1997, in The Dynamical Behaviour of Our Planetary System, ed. R. Dvorak & J. Henrard (Dordrecht: Kluwer), 345
Szebehely, V. 1967, Theory of Orbits (New York: Academic)
Wisdom, J., & Holman, M. 1991, AJ, 102, 1528