Statistics of Radioactive Decay

Introduction

The purpose of this experiment is to analyze a set of data that contains natural variability from sample to sample, but for which the probability distribution function (i.e. the Poisson Distribution) is well known. You will compare your measurements to theoretically derived functions, and test the compatibility of your measurements and the theory. The data are obtained by measuring the decay of a radioactive source using a Geiger counter.

Goals:

- gain familiarity with the specific form and use of the Poisson distribution
- gain experience with the use of a Geiger counter for measuring radioactive decay events

Theory

The disintegrations from a radioactive substance occur at random. If the number of counts is measured for a pre-specified time interval, and then the measurement is repeated over the same time interval, it is likely that that number will be different every time you perform a new measurement. It can be shown that in such cases of random decay the frequency distribution $P(n)$ describing the probability of observing *n* disintegrations in some time interval ***t*** is given by the Poisson distribution. This has been discussed in your lectures. The Poisson distribution is given by

$$P(n) = \frac{\mu^n e^{-\mu}}{n!}$$

where μ is the mean of the distribution.

The standard deviation σ of this distribution is given by: $\sigma = \sqrt{\mu}$

For large values of μ, the Poisson distribution approaches a Gaussian (normal) distribution:

$$P(n) = \frac{1}{\sqrt{2\pi\mu}} e^{-(n-\mu)^2/2\mu}$$

The Poisson Distribution – discussion

In previous experiments, we discussed random experimental errors which resulted from limitations of the experiment itself due to imperfections of apparatus or technique. There is another class of error which is due to the random nature of the process being studied, and this second category is the subject of our current experiment. This is often associated with "counting" experiments (an important category in modern day physics) and follows the rules of **Poisson Statistics**.

Consider an experiment to measure radioactive decay. One could count the number of decays in a series of (say) 10-s intervals. Since radioactive decays are random in time, the number of decays will not be repeatable between successive measurements. There will be a mean value, but rarely will any measurement actually produce this value. For example, suppose the true mean was 9 counts. Then due to the intrinsic natural variability, it may be possible to measure as low as only 1 or 2 counts, or perhaps as high as 20 counts, using the same experimental setup, because of the random nature of the process. The Poisson distribution gives you the probability $P(n)$ that you will obtain n counts when the mean count over many measurements is μ.

Notice that n is an integer in this distribution (*i.e.* you either count something or you don't).

Also, this distribution is very non-symmetric (skewed) when μ is small. This is very different to the distributions that you have normally met so far, for which you assumed you could have a + or – error with equal probability. Part of the reason that the distribution is skewed is that you can't have a negative number of counts. However, when μ is large (say > 30) the Poisson distribution approaches a Gaussian distribution with a mean μ and a standard deviation $\sqrt{\mu}$.

If you are doing a counting experiment in the laboratory (*e.g.* scattering, γ -ray spectroscopy, or α-particle absorption), you can say that the *best estimate* of the mean number of counts is the measured mean value $\bar{n}$, and the *best estimate* of the standard deviation about this mean is $\sqrt{\bar{n}}$. The point here is that you can never know the true mean μ because that would require an infinite number of measurements.

The Geiger-Muller (GM) Counter

The GM counter is a combination of two devices:

- a GM tube for the detection of radioactivity

- an instrument that amplifies and counts the electrical signals received from the GM tube

The tube has a very thin glass window facing the radioactive source. This permits the electrons emitted from the source to pass into the interior with minor absorption and scattering. The GM tube consists of an outer cylindrical conductor and a central wire maintained at a positive potential with respect to the outer cylinder. As an energetic particle crosses the detector it will ionize a gas molecule in the chamber. The resulting electron and ion are accelerated by the potential, each towards the electrode with opposite sign. For large enough potential difference the electron released in the ionization process will ionize another molecule and this will result in an avalanche of charges that could be detected as an electrical signal. The range of voltages applied to the GM tube that will generate a signal for an incoming ionizing particle is called the Geiger-Muller region. In other words the sensitivity of the amplifier connected to the tube is such that counts are only detected when the tube is operating in the Geiger-Muller region. Different tubes have different characteristics. In our experiment we will use a tube with a range of 300-1000V.

The radioactive source used in this experiment is Thallium 204, a radioactive isotope with a half-life of 3.8 years. When it disintegrates, an electron with energy of 0.77 MeV is emitted. It thus has sufficient energy to penetrate the tube and produce a count.

Experiment:

Apparatus:

- Geiger-Muller tube, stand and shelf
- High voltage source and counter
- Tl^{204} source
- timer

Procedure

Part I

Among your equipment you will find two semicircular pieces of plastic. One is unmarked; this is the non- radioactive blank. The other is the radioactive source. Record the number and letter on the source in your lab book.

Check that the high voltage controls are set at minimum values, and switch the power on. The “test” switch should be off, and the “count” switch should be on. Switch on the preset timer, and set it to 10 seconds. Put the source and the blank together on the tray and put it on shelf number 4.

Increase the voltage to 500 V. Start the timer and record the number of counts in 10 seconds. If necessary move the source to another shelf and repeat until the number of counts is between 800 and 4000. If you still cannot achieve a value of at least 800, record for a little longer, *e.g.* 15 or 20 or even 30 seconds.

Measure the number of counts in your chosen time interval. Repeat 45 times.

Part 2

Set the source far from the detector so that the number of counts obtained in 10 seconds is small. This will represent your "background level", and is due to natural radioactivity around you – in the walls, due to cosmic ray particles etc. Typically you should get values of the order of 1, 2, 3, 4, with less frequent occurrences of numbers like 5, 6, 7 etc. If you find you are getting lots of 5's and 6's, say, *reduce* the recording time to say 5 seconds. If you are getting mainly 0's and 1's, *increase* your recording time. Measure the number of counts in your selected recording period. Repeat 45 times. Note that the recording interval you choose does *not* have to be the same as in Part I.

Analysis

Analyze the "background counts" *first*, *then* the "source" results, as follows.

a) Background counts.

1) Plot the data in a histogram format (i.e. abscissa = number of counts, ordinate = number of measurements yielding that number of counts). Use a bin size of *one count*. You are encouraged to do this with a spread-sheet (like Excel) if possible.

2) Determine the mean, $\bar{n}$, and then plot the expected Poisson distribution for this mean value over top of your own data. Ensure that the area under your experimental curve and your theoretical curve are the same (remember that the formula given earlier for the Poisson distribution was normalized to unit area). Also check the standard deviation, and check if it is similar to $\sqrt{\bar{n}}$. (Note: this might be a good chance for you to learn how to use the "standard deviation" option on your calculator, or you might use the same function in your spread-sheet package. (It is STDEV in Excel)

3) You will have recorded 45 successive counts e.g. 2, 3, 2, 1, 4, 3, .. *etc*. Sum the points in groups of 3. In the above example, your first point will be 2+3+2 = 7, and the next point will be 1+4+3 = 8, etc. This will *simulate* using a longer averaging interval – e.g. if you used 10 second intervals, summing 3 successive values will give you an equivalent rate for a 30-second interval. You will now only have 15 points. Calculate your new mean and standard deviation. Check again whether the standard deviation is similar to $\sqrt{\bar{n}}$ (rigorous comparison is not necessary – a qualitative comparison will do for now). Plot the expected Poisson distribution, using your new mean. Comment on the symmetry of your graph relative to the first one. Also plot the theoretical distribution assuming it is a Gaussian.

b) Radioactive Source counts.

The case where you deal with the radioactive source can be considered as an extension of the "background case" to the situation of much higher count rates. What do you expect the distribution to start to look like at high count rates?

1. Plot the data in a histogram format (i.e. abscissa = number of counts, ordinate = number of measurements yielding that number of counts). Note that this time you will need to experiment with the best choice of bin-size. Describe why you use the choice of bin-size that you do. (Again, you might also find it useful to use a spreadsheet).

2. Determine the mean, and the standard deviation, and check that the relation $\sigma = \sqrt{\bar{n}}$ still seems valid. Then plot the expected theoretical distribution (for this mean value and standard deviation) over top of your own data. (Should this be a Poisson distribution, or a Gaussian, or are they equivalent? Discuss). Ensure that the area under your experimental curve and your theoretical curve are the same.

3. From **your histogram**, estimate the probability of getting a single number of counts *n* which lies between $\bar{n}$ and $\bar{n}+50$. Now repeat the same determination, but this time using your **theoretical fitted curve**. In this case you may assume that your standard deviation squared is an exact estimate of the true variance, so you may treat your distribution as a normal distribution with known variance. Remember you are looking at the likelihood of obtaining a *single* value in this range. Will you use the standard deviation σ, or $\sigma/\sqrt{45}$, in your calculations? You may use your "normal-distribution" table in your lecture notes if you like.

4. Divide the data into three groups (*a,b,c*) of 15 measurements, and find the mean and *its* standard deviation for the three sets. For example, for set *a*

   $$\bar{n}_a = \left(\sum_{i=1}^{15} n_i\right) \Big/ 15 \text{ and } \sigma_{\bar{n}_a} = \sqrt{\sum_{i=1}^{15}\left(n_i - \bar{n}_a\right)^2 \Big/ (15-1)(15)}.$$

   Test the hypothesis that these 3 data sets are statistically equivalent by determining whether the 3 means are really the same number, statistically speaking. Do this by computing the mean of the 3 means, $\bar{n}_{all}$, and the corresponding χ^2:

   $$\chi^2 = \frac{\left(\bar{n}_a - \bar{n}_{all}\right)^2}{\sigma^2_{\bar{n}_a}} + \frac{\left(\bar{n}_b - \bar{n}_{all}\right)^2}{\sigma^2_{\bar{n}_b}} + \frac{\left(\bar{n}_c - \bar{n}_{all}\right)^2}{\sigma^2_{\bar{n}_c}}.$$

   From the χ^2 distribution, the most likely value of χ^2 is 2. Why?
   If you have access to Excel (use CHIDIST) or MATLAB (or equivalent), you can get the probability that your value of $\chi^2 > 2$. If this probability is not too small, say greater than 10%, then you have shown that the hypothesis is true (statistically speaking of course!).