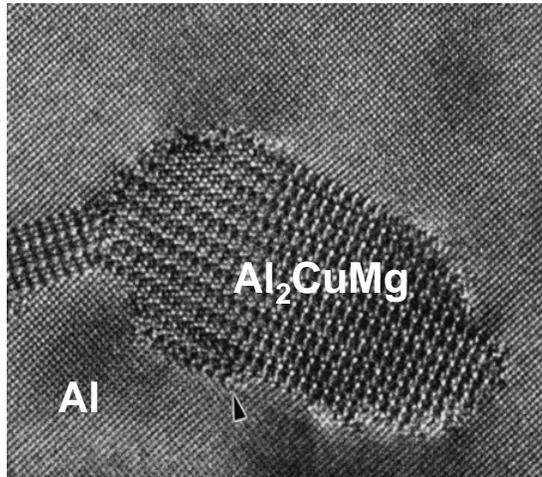


Chapter 8

Phase Diagrams

A **phase** in a material is a region that differ in its microstructure and or composition from another region



- homogeneous in crystal structure and atomic arrangement
- have same chemical **and** physical properties throughout
- have a definite **interface** and able to be mechanically separated from its surroundings

Chapter 8 in Smith & Hashemi

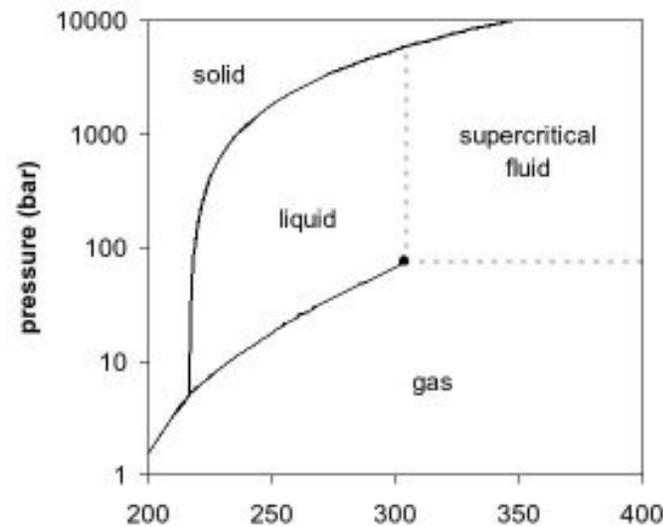
Additional resources: Callister, chapter 9 and 10

Phase diagram and “degrees of freedom”

A **phase diagrams** is a type of graph used to show the *equilibrium* conditions between the thermodynamically-distinct phases; or to show what phases are present in the material system at various T , p , and **compositions**

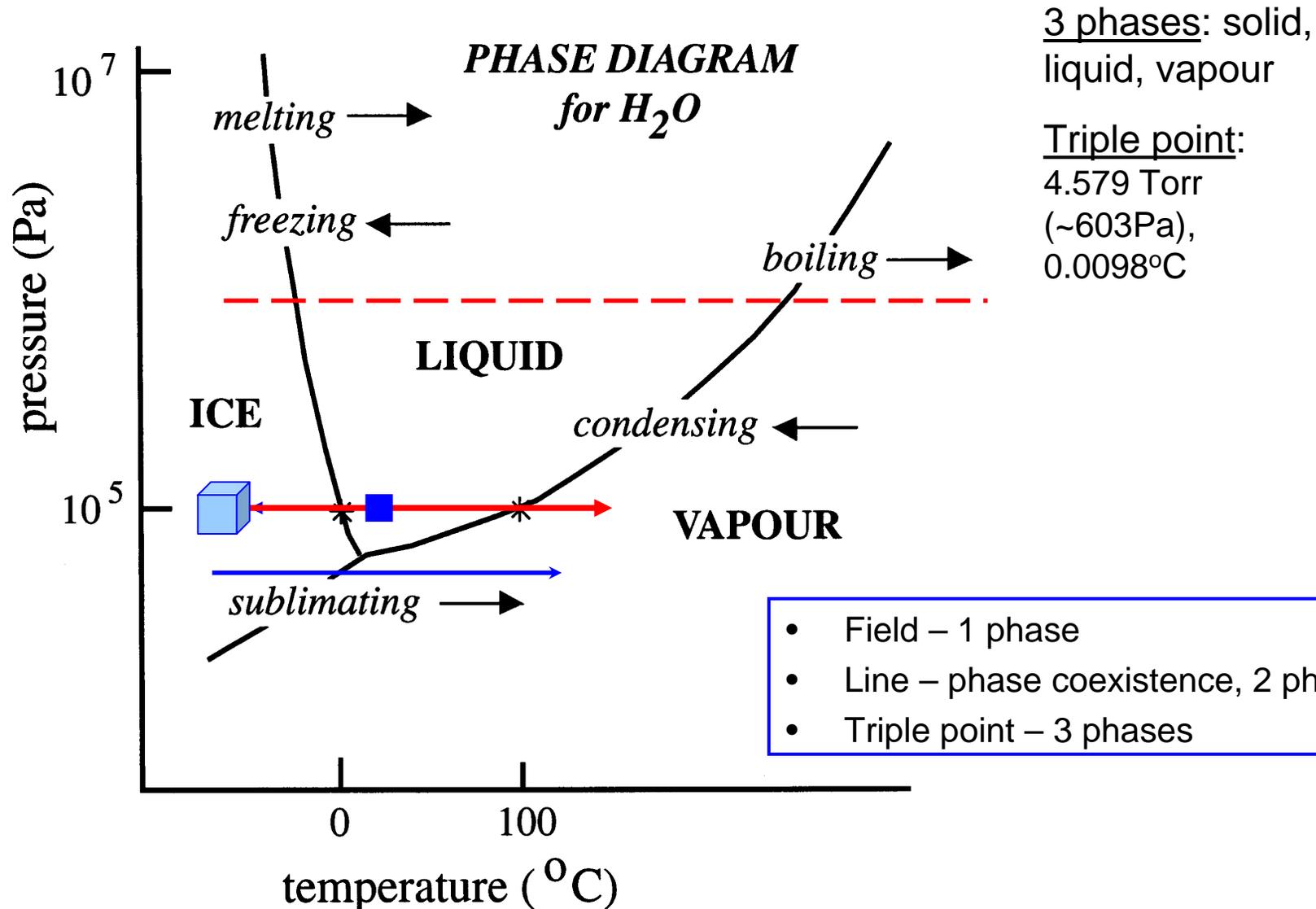
- “equilibrium” is important: phase diagrams are determined by using slow cooling conditions \Rightarrow no information about kinetics

Degree of freedom (or variance) F is *the number* of variables (T , p , and composition) that can be changed independently without changing the phases of the system



Phase diagram of CO₂ temperature (K)

8.1 Phase Diagram of Water



8.2 Gibbs Phase Rule

Gibbs' phase rule describes the possible # of **degrees of freedom (F)** in a **closed system** at **equilibrium**, in terms of the number of separate **phases (P)** and the number of **chemical components (C)** in the system (derived from thermodynamic principles by Josiah W. Gibbs in the 1870s)

$$F + P = C + 2$$

F is # of degrees of freedom or variance

P is # of phases

C is # of components

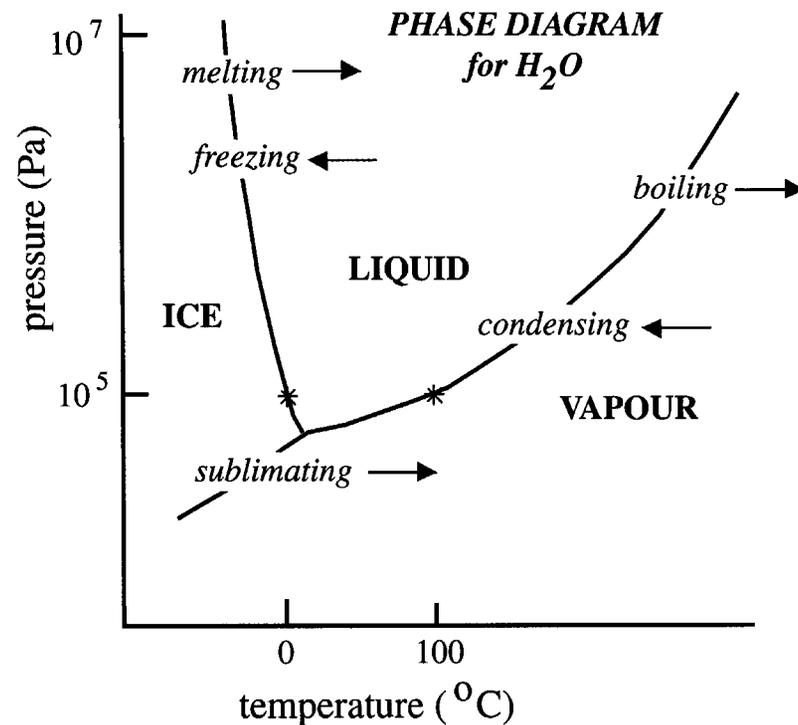
Component is the minimum # of species necessary to define the composition of the system

H₂O $C=1$

(i) $P=1, F=2$;

(ii) $P=2, F=1$;

(iii) $P=3, F=0$

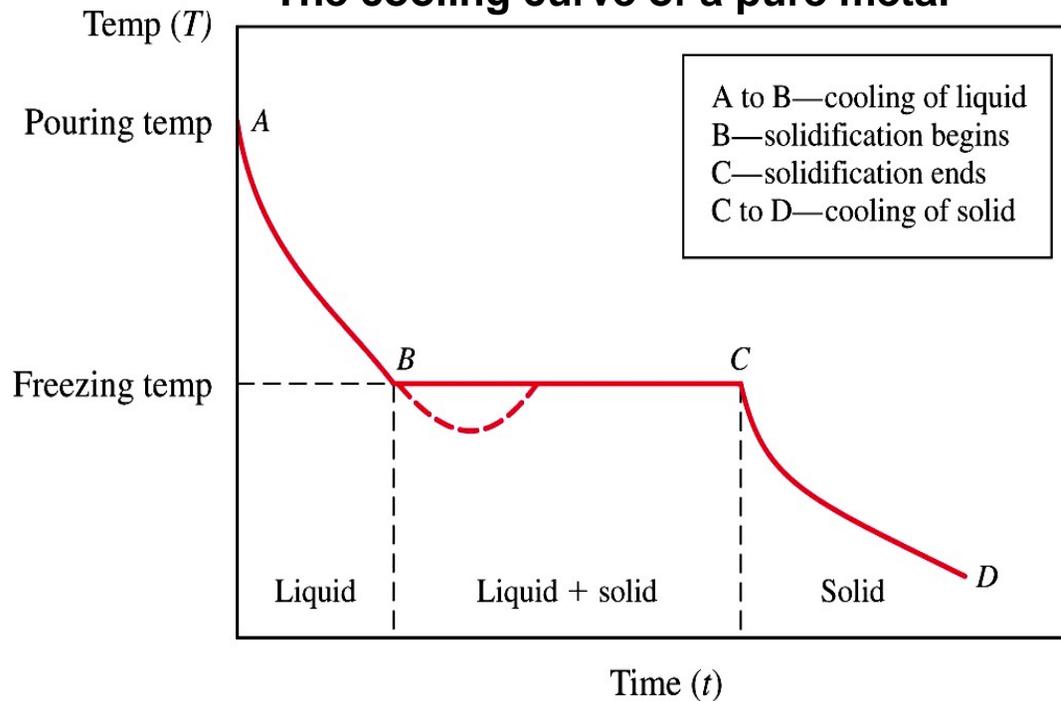


8.3 How to construct phase diagrams? - Cooling curves

Cooling curves:

- used to determine phase transition temperature
- record T of material vs time, as it cools from its molten state through solidification and finally to RT (at a constant pressure!!!)

The cooling curve of a pure metal

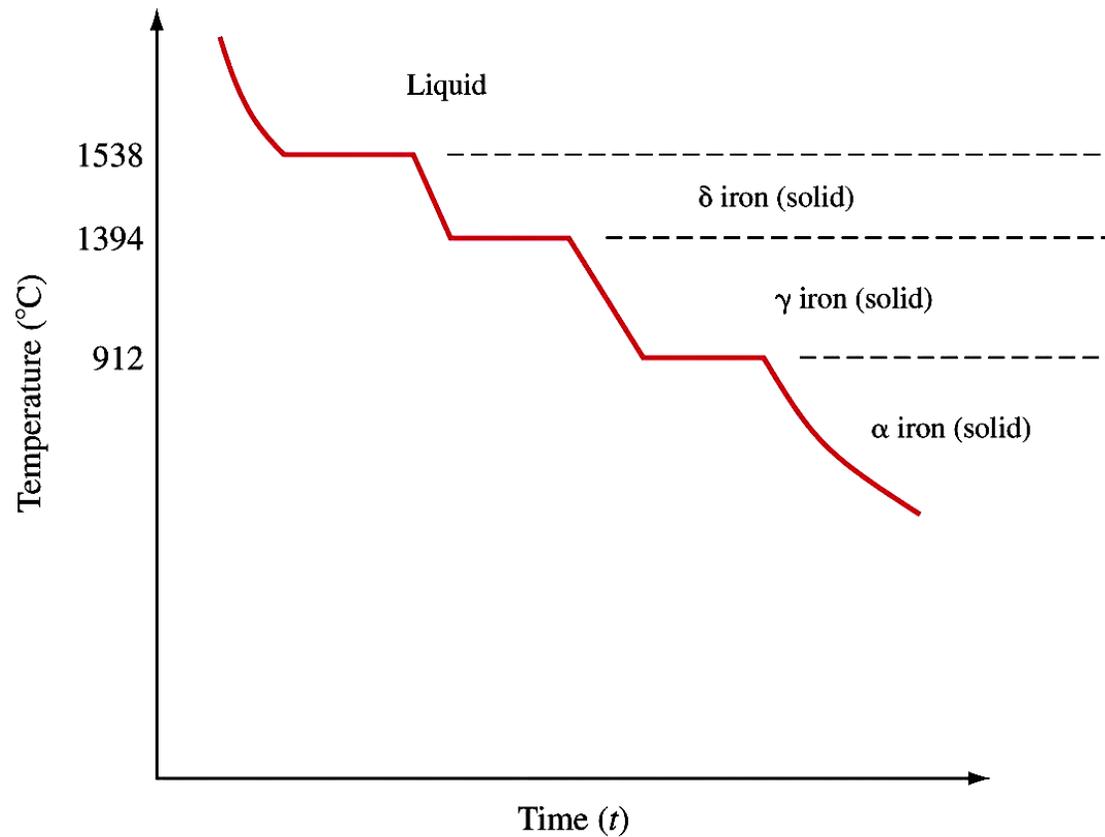


BC: *plateau* or *region of thermal arrest*, in this region material is in the form of solid and liquid phases

CD: solidification is completed, T drops

Cooling curve for pure iron @ 1atm

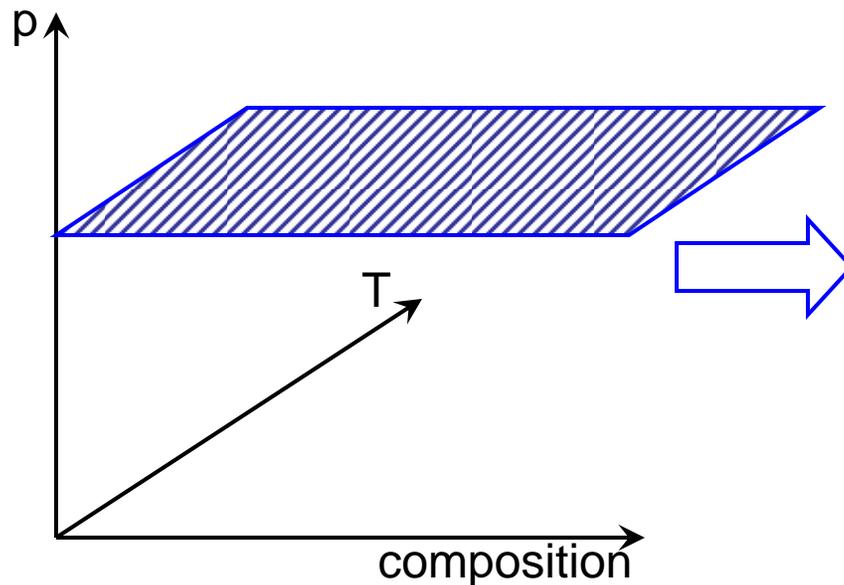
As $T \downarrow$: melted iron (liquid) \Rightarrow *bcc* Fe, δ (solid) \Rightarrow *fcc* Fe, γ (solid) \Rightarrow *bcc* Fe, α (RT)



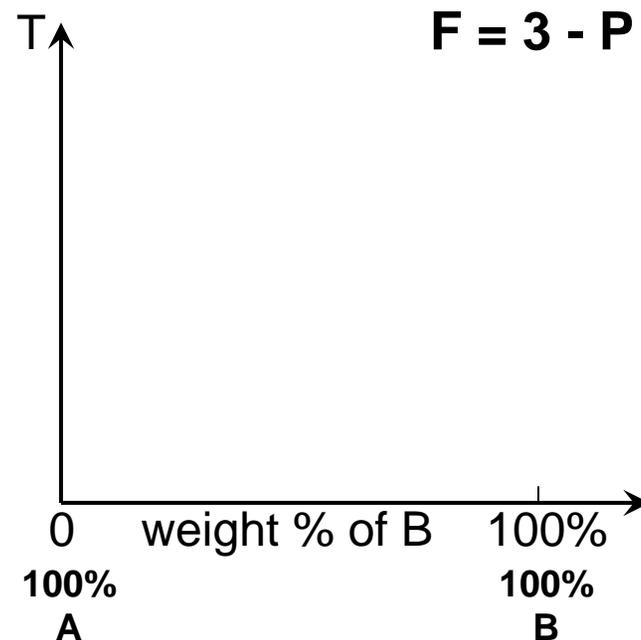
8.4 Binary systems (C = 2)

$$F + P = C + 2 = 4 \Rightarrow F = 4 - P$$

Degrees of freedom (F):
 p , T , composition



At $p = \text{const}$ (or $T = \text{const}$)



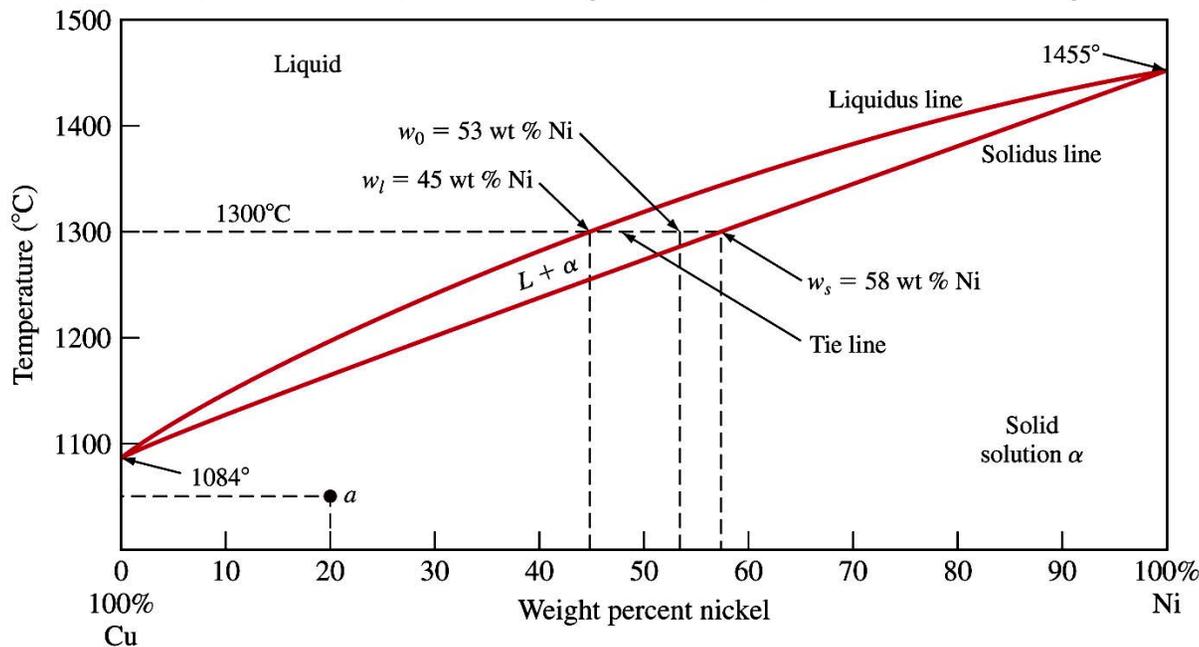
1. Two components are completely **mixable** in liquid and solid phase (form a solid state solution), and don't react chemically
2. Two components (A and B) can form **stable compounds** or alloys (for example: A, A_2B , A_3B , B)

Binary Isomorphous Alloy System (C=2)

Isomorphous: Two elements are completely soluble in each other in solid and liquid state; substitutional solid state solution can be formed; single type of crystal str. exist

Reminder: Hume-Rothery rules: (1) atoms have **similar radii**; (2) both pure materials have same crystal structure; (3) similar electronegativity (otherwise may form a compound instead); (4) solute should have higher valence

Example: Cu-Ni phase diagram (only for slow cooling conditions)

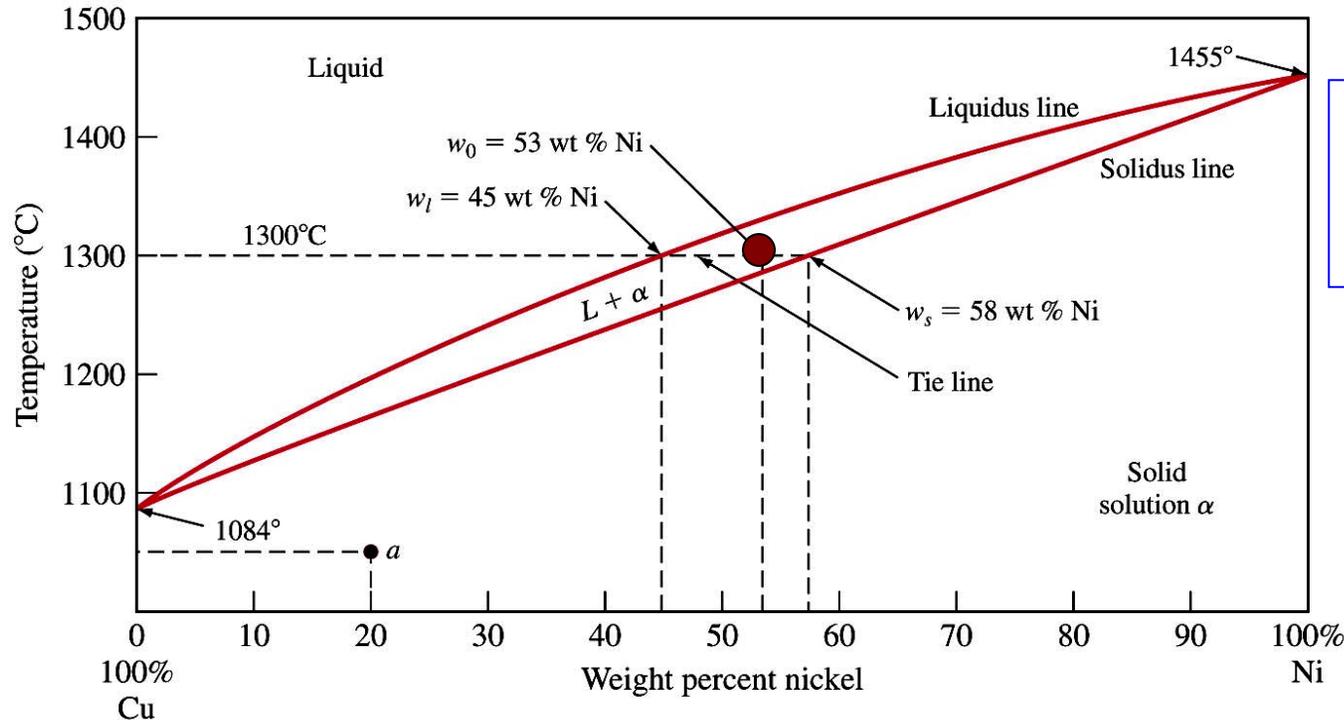


Liquidus line: the line connecting T_s at which liquid starts to solidify under equilibrium conditions

Solidus: the temperature at which the last of the liquid phase solidifies

Between liquidus and solidus: $P = 2$

53 wt% Ni – 47 wt% Cu at 1300°C



$$P = 1$$

$$F = 3 - P = 2$$

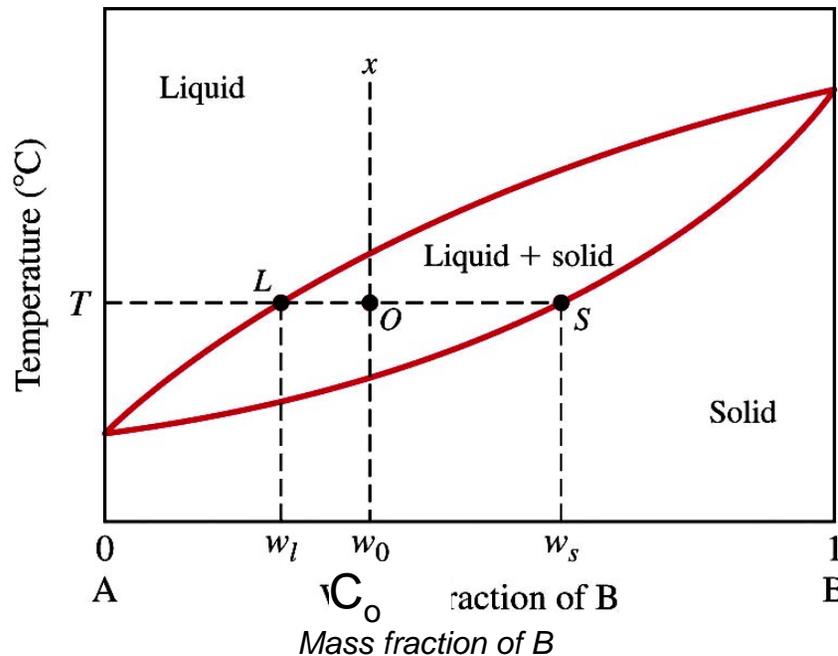
$$P = 2 ; F = 3 - P = 1$$

- contains both liquid and solid phases \Rightarrow neither of these phases can have average composition 53 wt% Ni – 47 wt% Cu
- draw a **tie line** at 1300°C \Rightarrow from the graph: composition of liquid phase $w_L = 45\%$ and solid phase $w_S = 58\%$ at 1300°C

8.5 The Lever Rule

The weight percentages of the phases in any 2 phase region can be calculated by using the **lever rule**

Consider the binary equilibrium phase diagram of elements A and B that are completely soluble in each other



Let x be the alloy composition of interest, its mass fraction of B (in A) is C_0

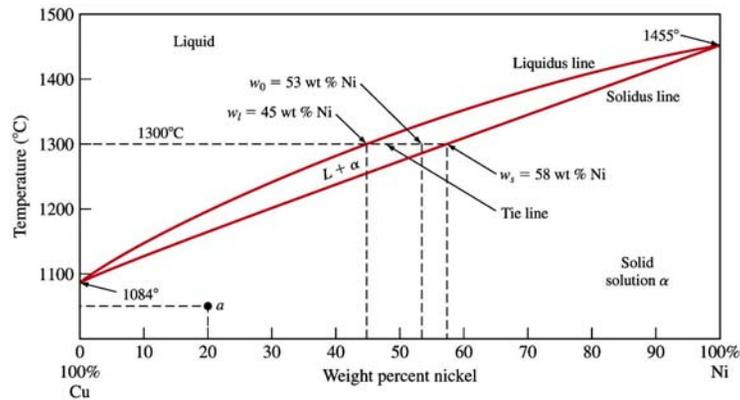
Let T be the temperature of interest \Rightarrow at T alloy x consists of a mixture of liquid (with C_L - mass fraction of B in liquid) and solid (C_S - mass fraction of B in solid phase)

Lever Rule (cont.)

Q.: A Cu-Ni alloy contains 47 wt % Cu and 53% of Ni and is at 1300°C. Use Fig.8.5 and answer the following:

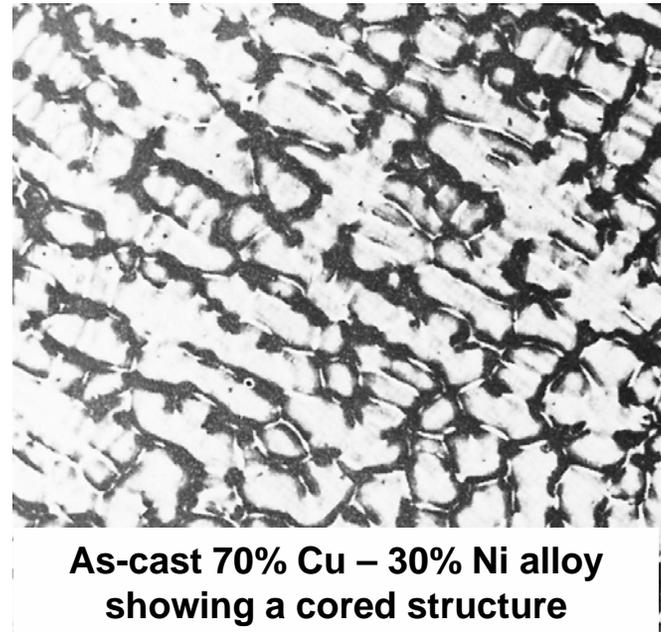
- A. What is the weight percent of Cu in the liquid and solid phases at this temperature?
- B. What weight percent of this alloy is liquid and what weight percent is solid?

8.6 Nonequilibrium Solidification of Alloys

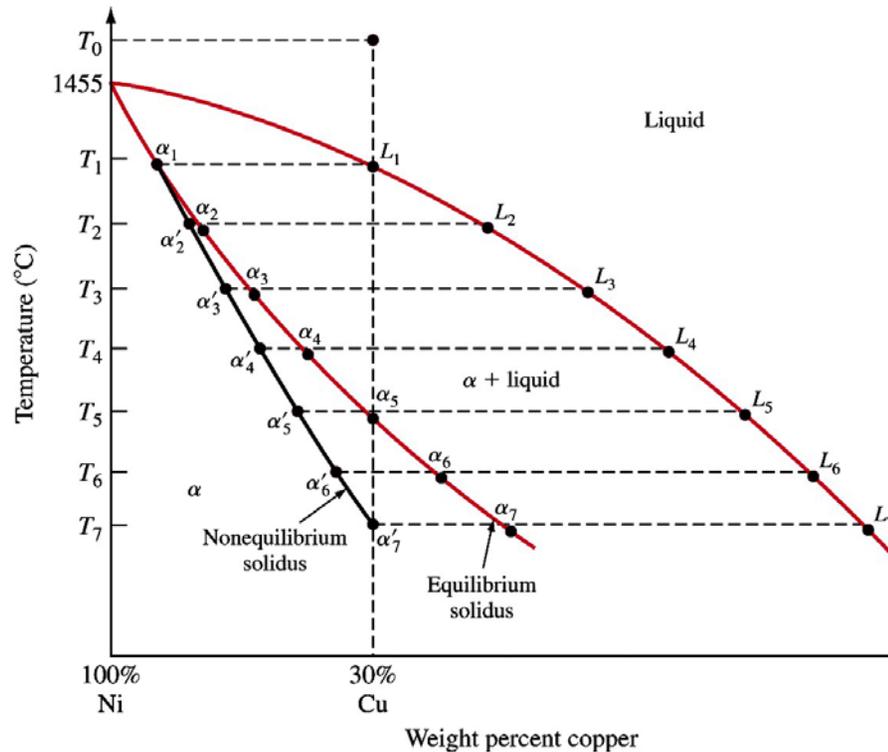


← constructed by using very slow cooling conditions

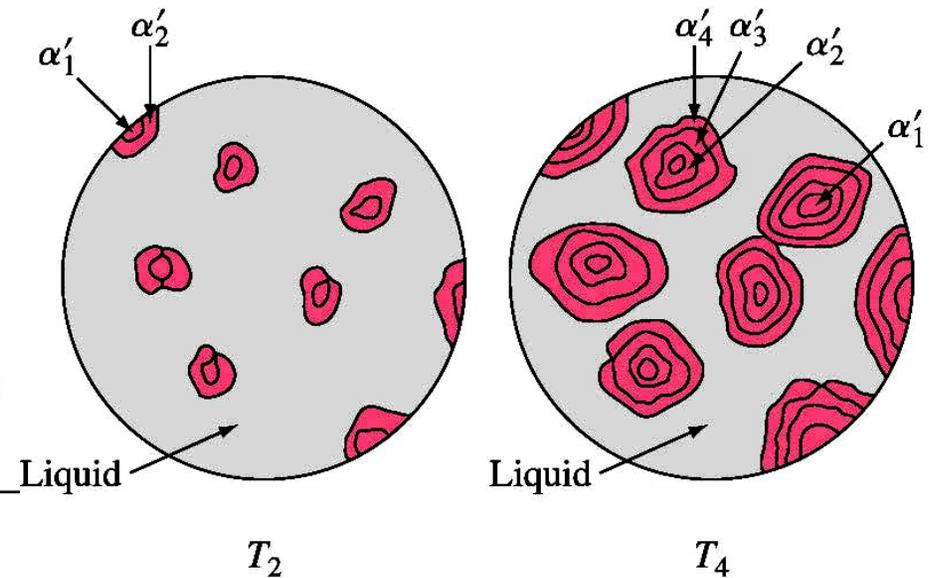
Atomic diffusion is slow in solid state; as-cast microstructures show “core structures” caused by regions of different chemical composition



Nonequilibrium Solidus



Solidification of a 70% Ni-30%Cu alloy
Fig. 8.9, Smith

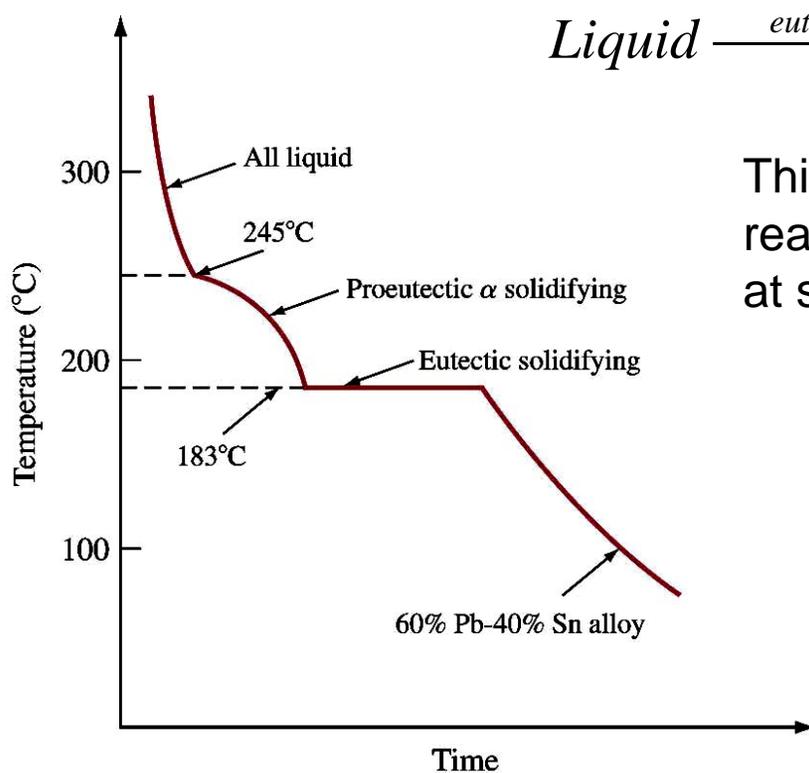


Schematic microstructures at T_2 and T_4
Fig.8.10, Smith

- each core structure will have composition gradient α_1 - α_7
- additional **homogenization** step is often required (annealing $<T_7$)

8.7 Binary Eutectic Alloy System

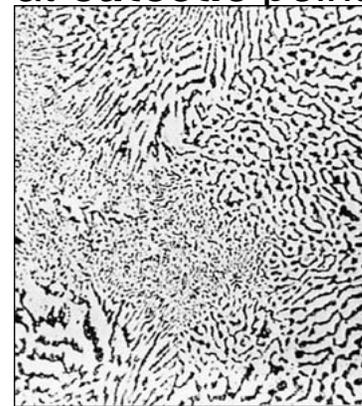
- Components has **limited** solid solubility in each other
- Example: cooling 60%Pb – 40%Sn system



This *eutectic* reaction is called an *invariant* reaction \Rightarrow occurs under equilibrium conditions at specific T and alloy composition

$$F=0$$

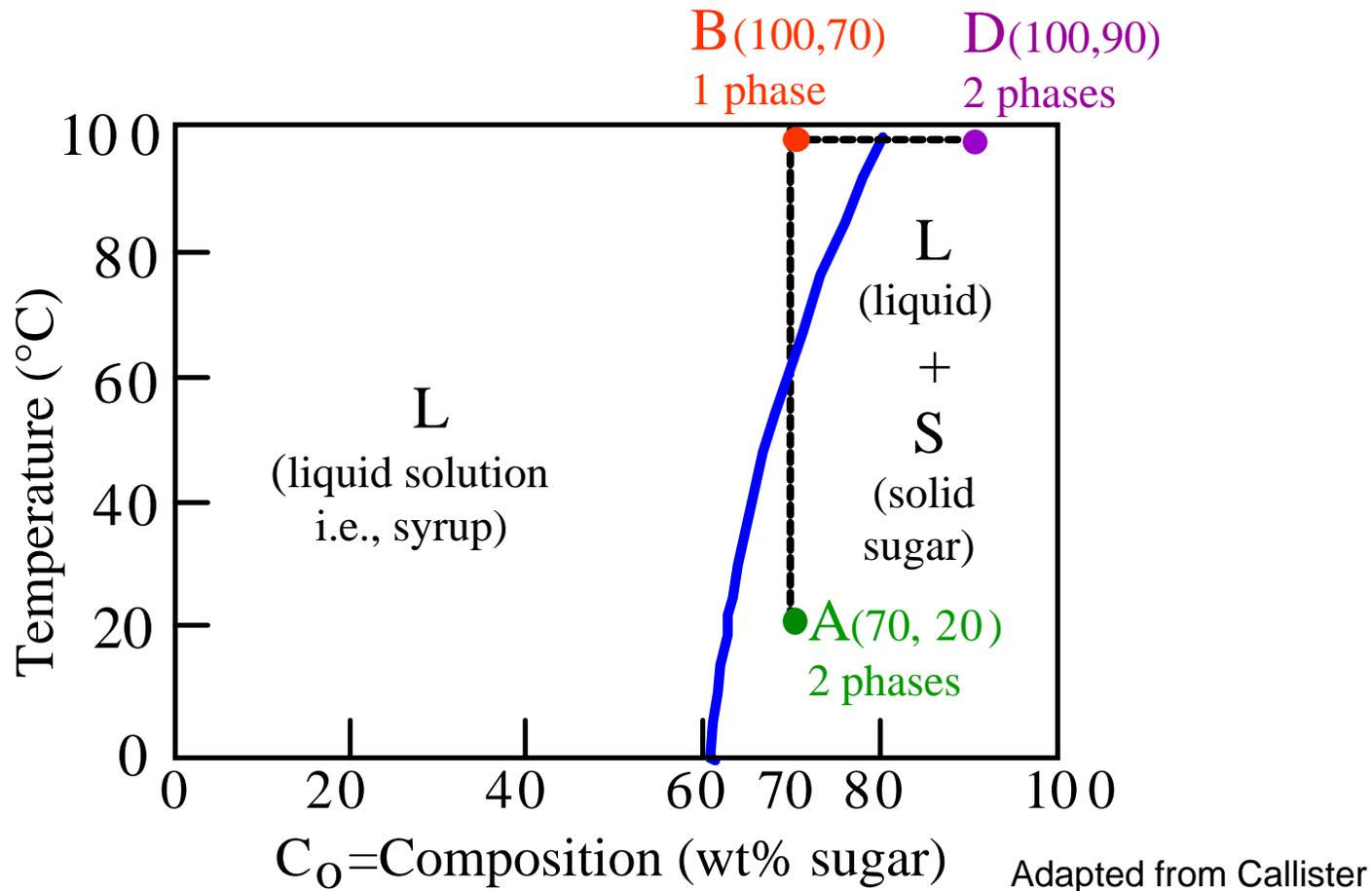
at **eutectic** point



(a)

Solubility Limit: Water-Sugar

- Changing T can change # of phases: path A to B.
- Changing C_o can change # of phases: path B to D



Binary Eutectic Alloy System

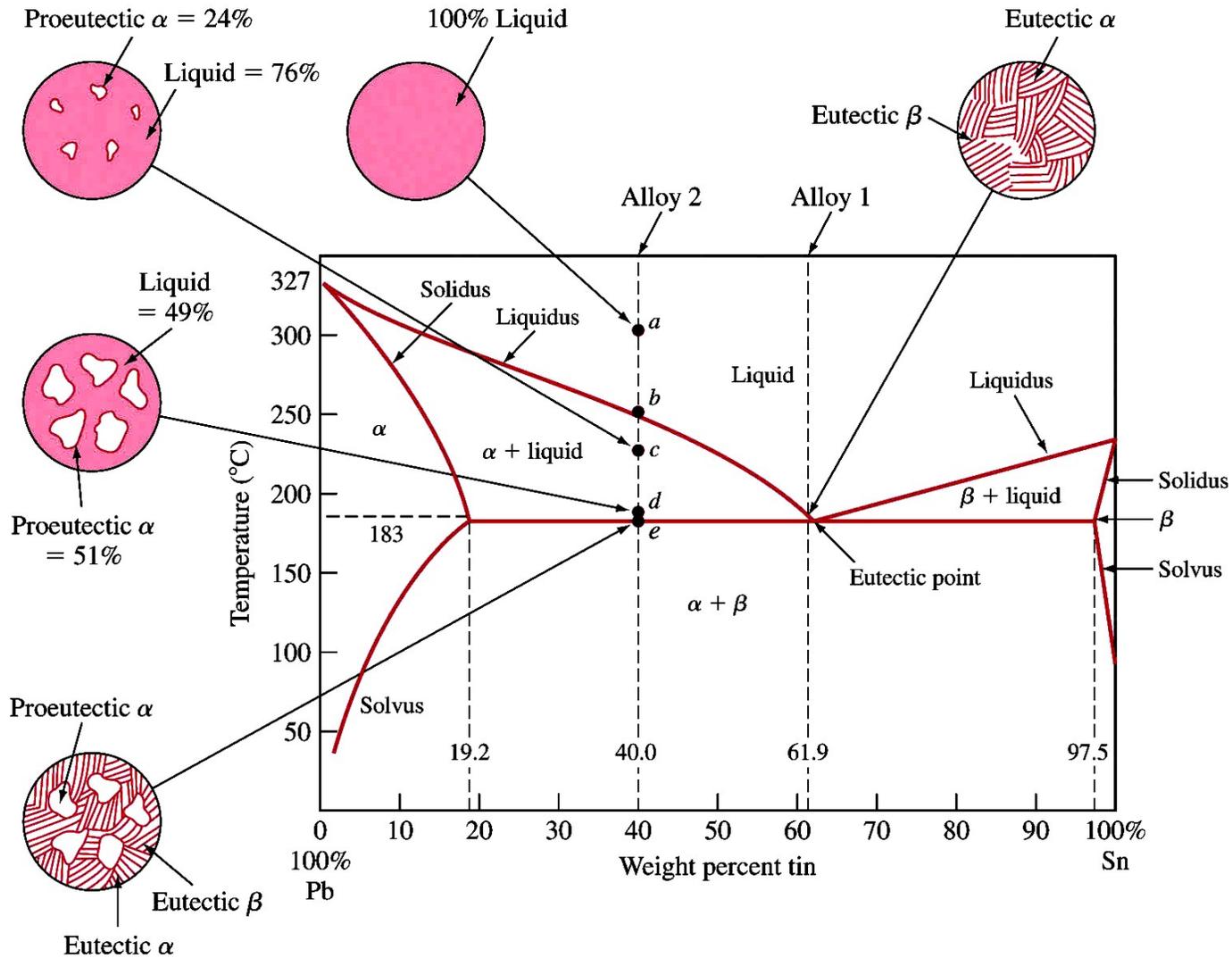


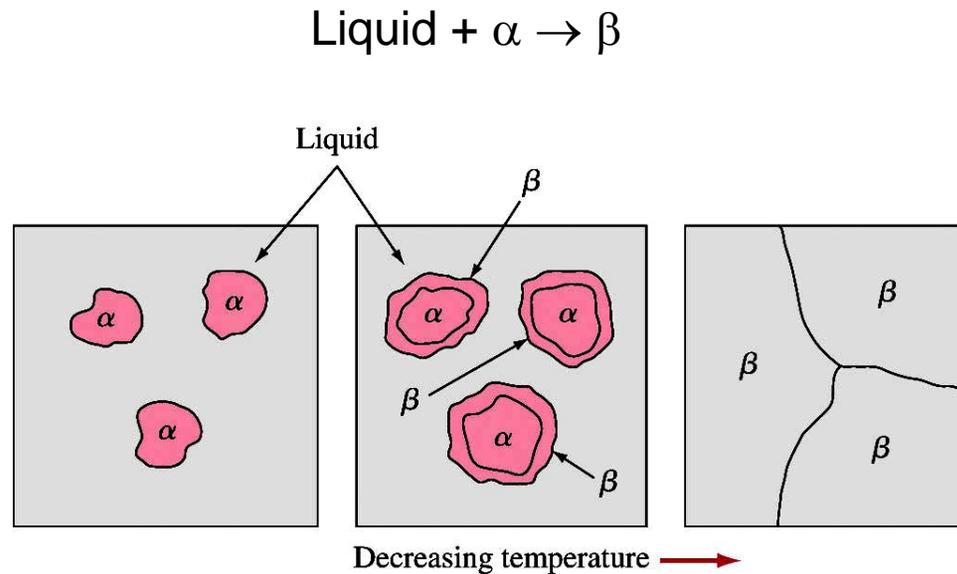
Figure 8.13, Smith

Q: A lead-tin (Pb – Sn) alloy contains 64 wt % proeutectic (α) and 36% eutectic $\alpha+\beta$ at $183^\circ\text{C} - \Delta T$. Using Figure 8.13, calculate the average composition of this alloy.

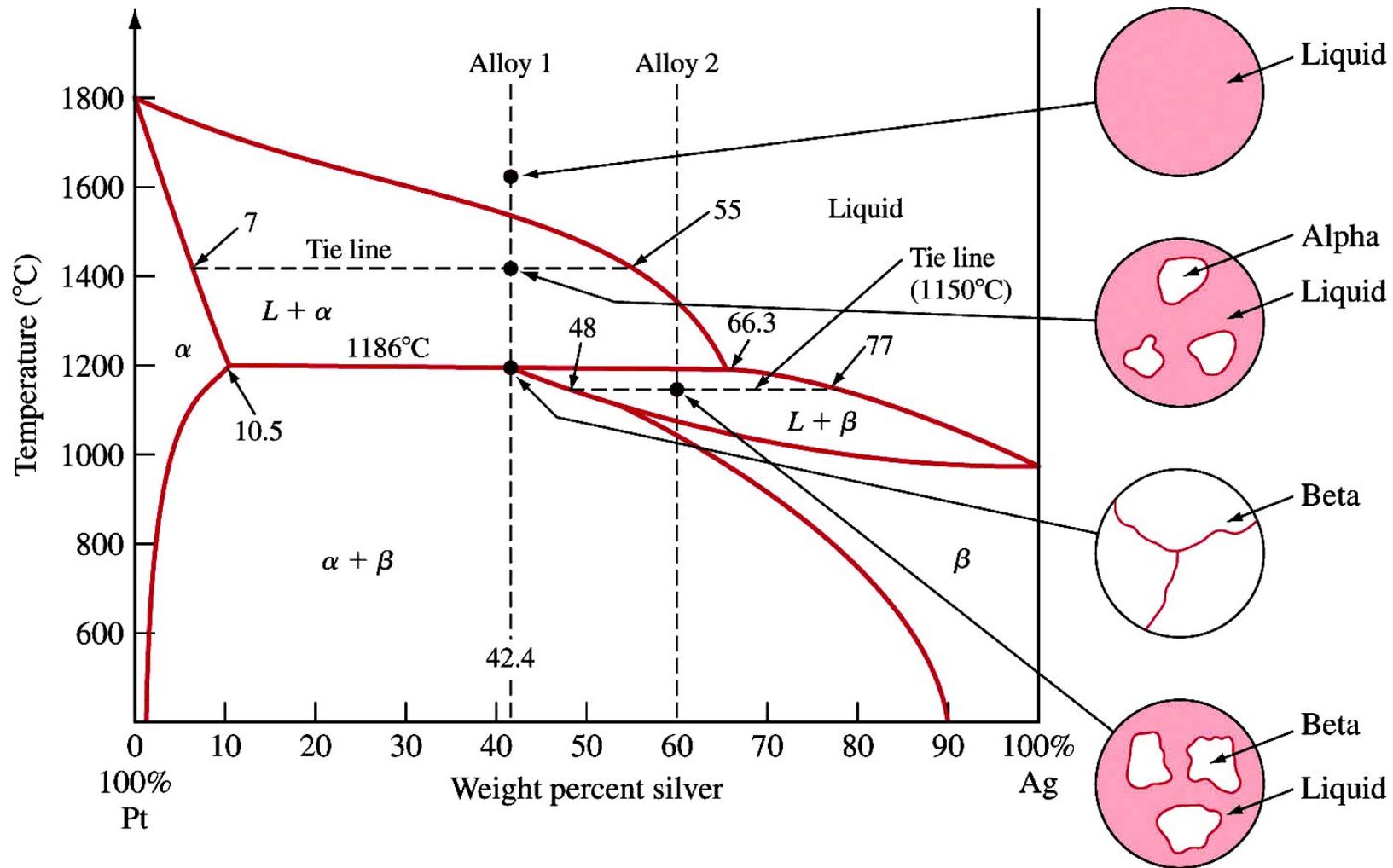
8.8 Binary Peritectic Alloy System

The melting points of the two components are quite different

A liquid phase reacts with the solid phase to form a new and different solid phase

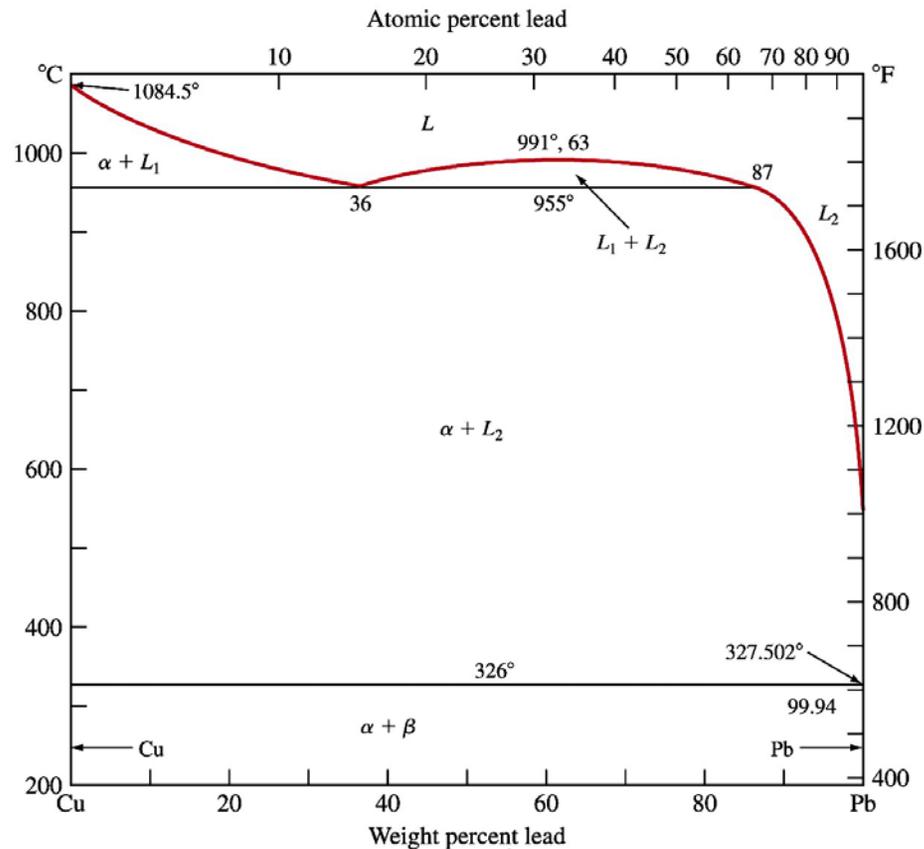
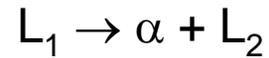


Binary Peritectic Alloy System (cont.)



8.9 Binary monotectic systems

Monotectic reaction: a liquid phase transforms into a solid phase and another liquid phase



8.10 Invariant Reactions

To summarize:

5 invariant reactions ($F = 0$)

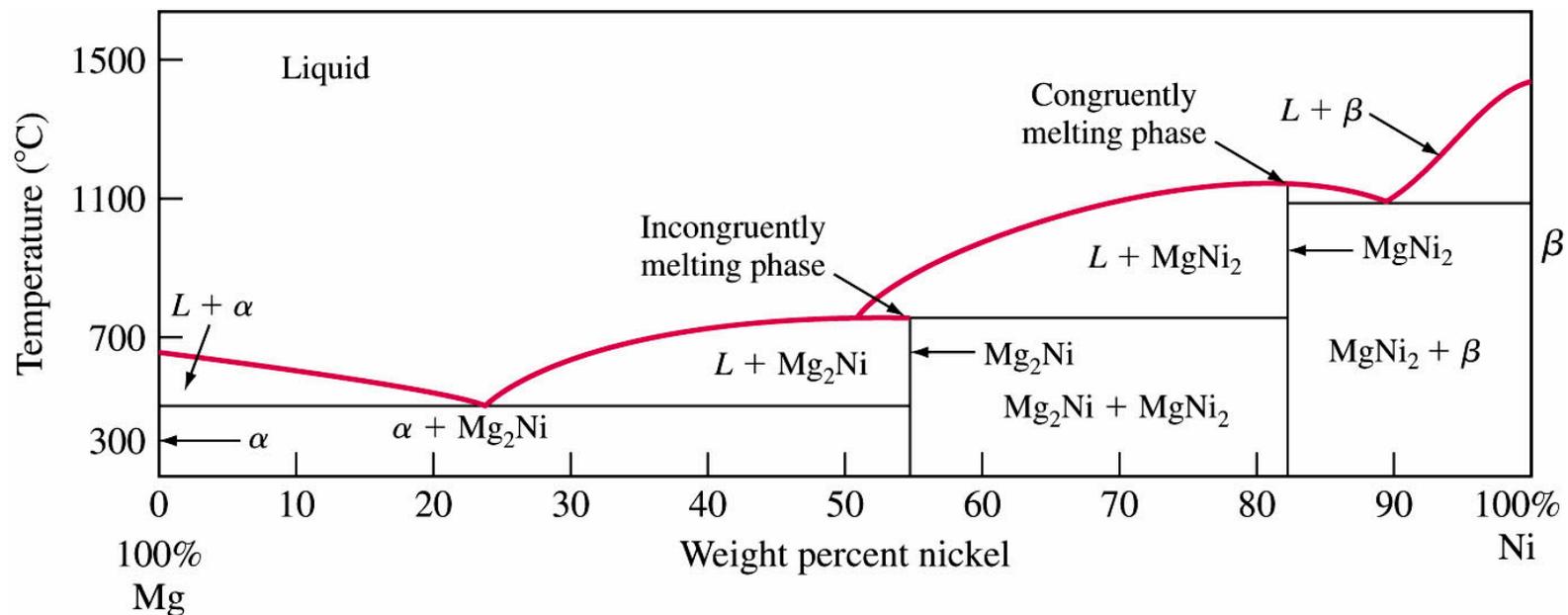
- | | | |
|----------------|-------------------|------------------------------|
| 1. Eutectic | Liquid | $\rightarrow \alpha + \beta$ |
| 2. Eutectoid | α | $\rightarrow \beta + \gamma$ |
| 3. Peritectic | Liquid + α | $\rightarrow \beta$ |
| 4. Peritectoid | $\alpha + \beta$ | $\rightarrow \gamma$ |
| 5. Monotectic | L_1 | $\rightarrow \alpha + L_2$ |

The eutectic and eutectoid reactions are similar in that they both involve the decomposition of a single phase into two solid phases. The *-oid* suffix indicates that a solid, rather than liquid, phase is decomposing.

8.11 Phase Diagrams with Intermediate Phases and Compounds

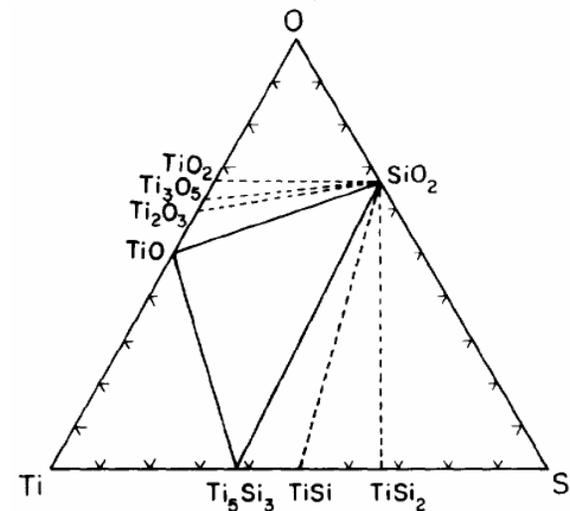
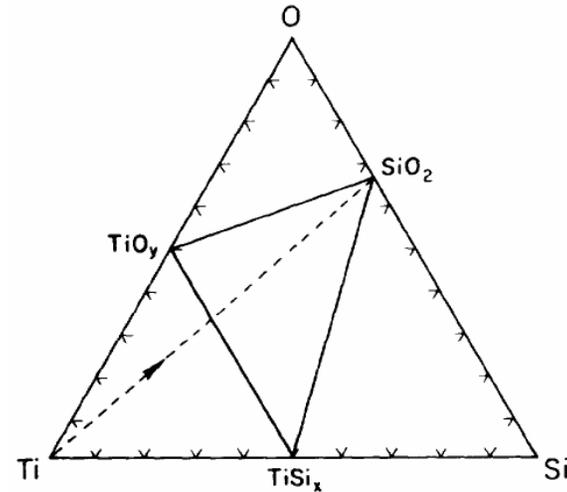
Terminal phase: a solid solution of one component in another for which one boundary of the phase field is a pure component

Intermediate phase: a phase whose composition range is between those of terminal phases



Ti-Si-O system

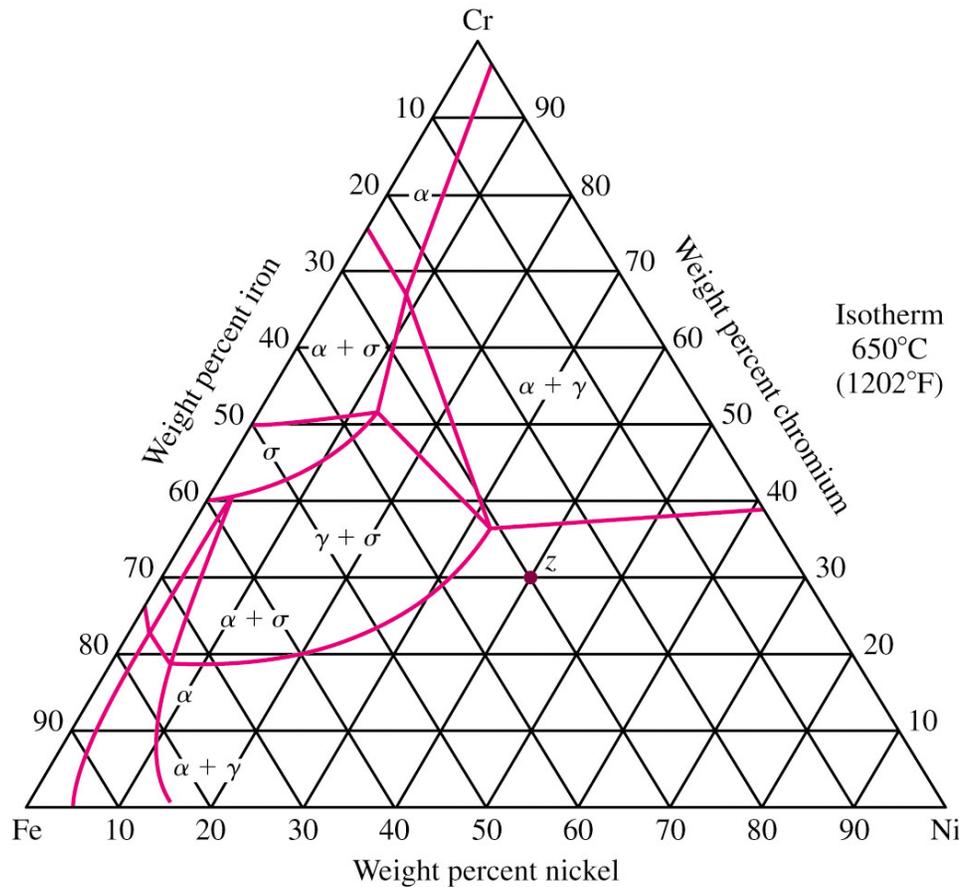
- Experiment (700-1000°C)
 $\text{Ti} + \text{SiO}_2 \rightarrow \text{Ti}_5\text{Si}_3$ and TiO_y
- At equilibrium the system will be in $\text{TiSi}_x - \text{TiO}_y - \text{SiO}_2$ three phase region (from calculations)
- $\text{Ti}_5\text{Si}_3 - \text{TiO} - \text{SiO}_2$ three phase region determined experimentally and remaining tie lines can be inferred



8.12 Ternary Phase Diagram

$$F + P = C + 2$$

For $p = 1\text{atm}$, $T = \text{const}$ (isotherms)



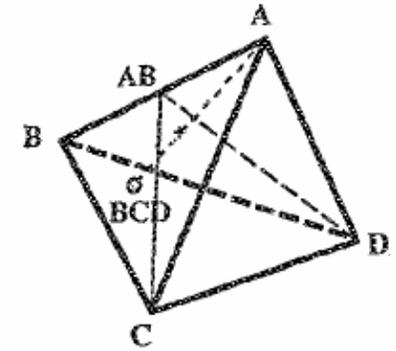
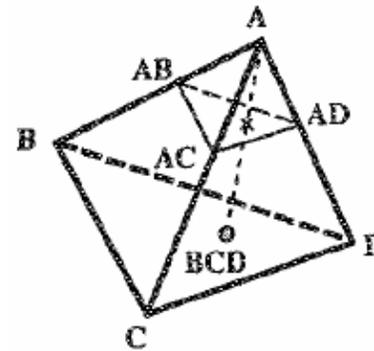
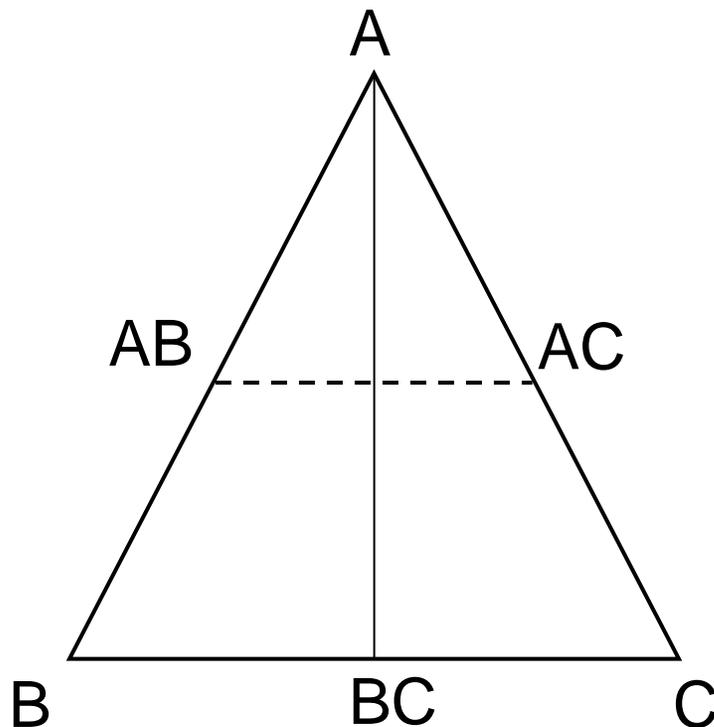
Cr-Fe-Ni alloy
stainless steel

Three and four component system

$$AB + AC = 2A + BC$$

$$\Delta G = (2G_A + G_{BC}) - (G_{AB} + G_{AC})$$

If $\Delta G < 0$, there is tie line between A and BC
The remaining tie lines cannot cross



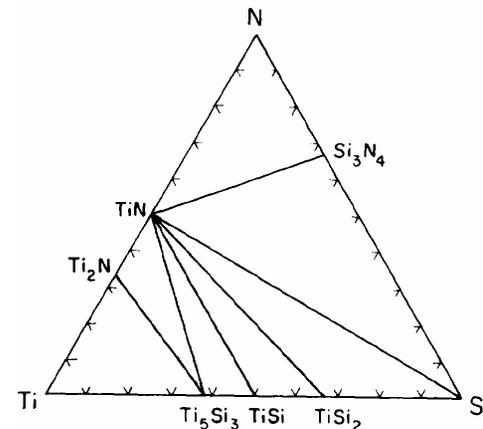
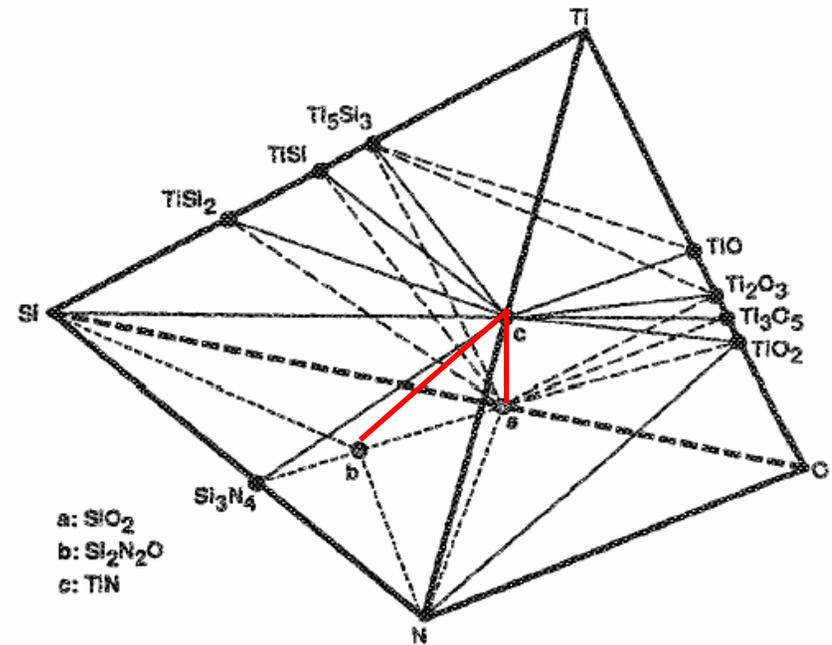
$$AB + AC + AD = 3A + BCD$$

$$\Delta G = (3G_A + G_{BCD}) - (G_{AB} + G_{AC} + G_{AD})$$

- Two phase equilibrium is represented by a tie line
- If $\Delta G < 0$, there is a tie line between A and BCD;
- otherwise plane connects AB-AC-AD

The Ti-Si-N-O quaternary phase diagram

- Entire phase diagram can be calculated by taking into account all possible combinations of reactions and products
- 4 ternary diagrams of Ti-Si-N, Ti-N-O, Ti-Si-O and Si-N-O were evaluated
- additional quaternary tie lines from TiN to SiO_2 and $\text{Si}_2\text{N}_2\text{O}$
- stable metallization bilayer of TiN and TiSi_2 in contact with SiO_2



A.S.Bhansali, *et al.*, *J.Appl.Phys.* 68(3) (1990) 1043

Z.Chen, *et al.*, *Phys.Stat.Sol.B* 241(10) (2004) 2253