

THE ZEEMAN EFFECT

PHYSICS 359E

INTRODUCTION

The Zeeman effect is a demonstration of spatial quantization of angular momentum in atomic physics. Since an electron circling a nucleus is analogous to a current flowing in a loop of wire, an atom would be expected to have a magnetic dipole moment

$$\mu_L = -(e/2)(\mathbf{r} \times \mathbf{v}).$$

Since the angular momentum of the orbit is given by $\mathbf{l} = \mathbf{r} \times \mathbf{p} = m(\mathbf{r} \times \mathbf{v})$,

$$\mu_L = -(e/2m)\mathbf{L}.$$

If an external magnetic field is present, a torque is exerted on which causes the magnetic moment to precess about the field lines. The energy associated with the interaction of μ_L with $\mathbf{B}$ is given by

$$\Delta E = -\mu_L \cdot \mathbf{B} = (e/2m)\mathbf{L} \cdot \mathbf{B},$$

which is clearly angle-dependent. If all orientations of $\mathbf{L}$ with respect to $\mathbf{B}$ were allowed, then a continuous range of E would occur and an atomic transition would show a spectral line broadened by an amount proportional to the field strength. Instead such spectral lines are observed to split into several *discrete lines*, indicating that only certain values of $\mathbf{L} \cdot \mathbf{B}$ are allowed; this implies spatial quantization of $\mathbf{L}$.

In general, an atom has an additional magnetic moment due to the intrinsic spin angular momentum $\mathbf{S}$ of the electrons. This additional spin angular momentum also has a magnetic moment, but with a different relationship between $\mathbf{S}$ and the magnetic moment of the electron:

$$\mu_S = -(e/m)\mathbf{S}.$$

Notice the factor of two difference. Taking both magnetic moments into account we expect the interaction energy to be

$$\Delta E = -\mu \cdot \mathbf{B} = -(\mu_L + \mu_S) \cdot \mathbf{B} = -(e/2m)(\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B}.$$

Thus the interaction energy is *not* simply proportional to the total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$, but to a different combination of $\mathbf{L}$ and $\mathbf{S}$. This creates a significant amount of extra bookkeeping.

Historically, the case $\mathbf{S} = 0$ is referred to as the normal Zeeman effect, while the general (and more common) case is known as the anomalous Zeeman effect. Both cases are discussed clearly in Refs. 1 and 2. The essential result you need to understand is that when a field B is present, a single atomic energy level characterized by quantum numbers S , L , and J splits into $2J + 1$ states of energy differing from the initial energy level by

$$\Delta E = \mu_B g B m_j$$

where $\mu_B = e\hbar/2m$ is the Bohr magneton, $-J \leq m_j \leq J$ is the projection of $\mathbf{J}/\hbar$ onto the field axis, and

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

is known as the Landé g factor. (You may find pp 33 – 43 in Ref. 1, or pp 348 – 364 in Ref. 2, a useful introduction to the notation used to describe atomic energy levels.)

PRE-LAB PREPARATION

1. Read carefully pp 215 – 228 in Ref.1 on the theory of the Zeeman effect. In your reading, note the definition and significance of the Landé g -factor, and the relationship between selection rules and polarization of the Zeeman transitions. Work out expressions for the energies of the photons emitted in all possible transitions in the following three cases, which you will study in this lab:

- the $2s^1S_0 - 3p^1P_1$ line of He at 501.5 nm (3 transitions, 3 distinct energies)
- the $6s6d^1D_2 - 6s6p^1P_1$ line of Hg at 579.1 nm (9 transitions, 3 distinct energies)
- the $6s7s^3S_1 - 6s6p^3P_2$ line of Hg at 546.1 nm (9 transitions, 9 distinct energies)

Here we describe the upper and lower atomic energy levels involved in each transition by first specifying the *electronic configuration* (e.g. $6s6d$ means that all shells through $4f$ and $5d$ are filled, and one electron is in each of the $6s$ and $6p$ shells), and then specifying a *specific state* of this configuration by giving the quantum numbers for total spin S , total orbital angular momentum L , and total angular momentum J of that state in the form $^{(2S+1)}L_J$, e.g. $^3P_2$.

Express the transition energies in terms of $\mu_B B$, where μ_B is the Bohr magneton, and B is the magnitude of the applied magnetic field. Evaluate the wavelength shifts in nm for these cases for a field of 0.5 T = 5000 G, draw diagrams of the level splitting and the resulting π and σ components of the spectral lines, and include these results in your report.

2. The Fabry-Pérot interferometer is a very important optical device because of its ability to resolve light of very small wavelength difference; it is also the basis of many laser cavities. As a general introduction to the theory of the Fabry-Pérot interferometer, go back to the lab write-up on the isotope effect in H from first semester. Be sure to read pp 172 – 177 in Ref. 4, which discusses its use in measuring small wavenumber differences. And remember: the length calibration on the micrometer barrel of the the Fabry-Perot does *not* read directly the spacing between the mirrors.

In what follows, the *wavenumber* is represented by the symbol $\bar{\nu}$ and is related to the wavelength λ by $\bar{\nu} = 1/\lambda$. In order to analyze the data from the manually operated interferometer, you will need to understand the concept of overlapping orders when light of more than one wavenumber is present. If two wavenumbers $\bar{\nu}_1$ and $\bar{\nu}_2$ are very close together, the two fringe patterns will coincide closely and each fringe will appear slightly broadened. As $|\bar{\nu}_1 - \bar{\nu}_2|$ increases, two separate fringe patterns will appear and eventually a fringe of order $n + 1$ and wavenumber $\bar{\nu}_2$ overlaps a fringe of order n and wavenumber $\bar{\nu}_1$ ($\bar{\nu}_2 > \bar{\nu}_1$). From the basic equation of the interferometer, $\bar{\nu} = n/(2t \cos \theta)$, you should show that when overlap occurs, $\bar{\nu}_2 - \bar{\nu}_1 \approx 1/(2t)$, assuming near normal incidence. This quantity is known as the *free spectral range* of the interferometer: $\Delta\bar{\nu}_{\text{fsr}} = 1/(2t)$.

In the case of the Zeeman splitting, as the field B is increased from zero, a $\Delta m_j = 1$ line from one order eventually overlaps with a $\Delta m_j = -1$ line from an adjacent order. You should convince yourself that each line has moved by $\Delta\bar{\nu}_{\text{fsr}}/2$. (Sometimes this condition is described in terms of a shift of the fringe pattern by ‘half an order’.)

PROCEDURE

In this experiment you can observe both the normal and anomalous Zeeman effects. The normal Zeeman effect can be observed in the He 501.6 nm and Hg 579.0 nm lines, and the anomalous Zeeman effect in the Hg 546.1 nm line. For each of these lines the line splitting can be measured as a function of magnetic field, and the value of the Bohr magneton can be determined.

The research aluminum foil magnet will produce a field of up to 8 kilogauss (0.8 T) with flat pole pieces and up to 20 kilogauss (2 T) with tapered pole pieces. A current of up to 9 A may be used. You should determine the homogeneity of the field over the region of the discharge tube, and consider how much of this region you should be observing. When you set up the interferometer, *be sure that you are observing the homogeneous region.*

Two different Fabry-Pérot interferometers will be used. One of these has a mirror separation of the order of one centimetre. The separation is varied manually, and the fringes are observed in a telescope. The arrangement of the apparatus is shown in Fig.1. The polarizer is used to isolate different sets of Zeeman lines.

The second interferometer is similar, but the plate spacing is smaller, and the separation is controlled electronically. With this apparatus a camera is available to photograph the interference fringes.

a) Manually operated interferometer

The manually operated interferometer should be used to measure the line splitting with the 501.6 nm He line in the following manner:

1. Set the mirrors of the étalon to a spacing of between 0.5 and 1.5 cm. (Spacers, removable by loosening two thumb screws, allow big changes in spacing.)
2. Align the mirrors with a laser by observing the transmitted beam on a screen and looking for interference. To perform the fine alignment, diverge the laser beam with a lens or tissue to fill the étalon aperture, observe the fringe pattern with the telescope, and adjust the mirror tilt *very carefully* for sharpest fringes.
3. Then send light from the He lamp through the interferometer, choosing a lens arrangement that makes the fringes as bright as possible. Readjust the tilt of the interferometer mirrors once more for the sharpest fringes with the darkest possible rings in between.
4. Insert a polarizer to transmit only the $\Delta m_j = \pm 1$ lines, and the appropriate interference filter to isolate the 501.6 nm line. (Note that the optics must be arranged to give as bright as possible a ring pattern, so that you can see it even with the filter in place!)
5. Observe the fringe pattern as you increase the field. You should see the fringe pattern blur out as the field increases, and then sharpen up as the fringe patterns overlap. Measure the field strength at which sharpening of the overlapped lines is clearest. If the tapered pole pieces are used, the current in the magnet should be recorded and the field measured later. (Note that the field strength produced by the magnet is *not* a linear function of the applied current. Why? Also be warned that one of the gaussmeters has a probe whose output reads 1/10th of the actual field strength. Always check your probe with one of the calibration magnets.)
6. Measure the mirror spacing with the vernier calipers.

Repeat the above steps for three mirror separations, using 0, 1 and 2 spacers. Your measurements may be limited by the difficulty of producing a visible fringe pattern at large mirror separations, by the limits of the magnet, and by the extinguishing of the discharge at large fields.

[Two sets of pole pieces are provided. The flat pole pieces should be used at low fields to give a uniform field while the tapered pole tips will be required to produce large fields.]

Plot $\Delta\bar{\nu}$ vs. B to find e/m and μ_B . Be sure to use the point for $B = 0$! Include this analysis in your report.

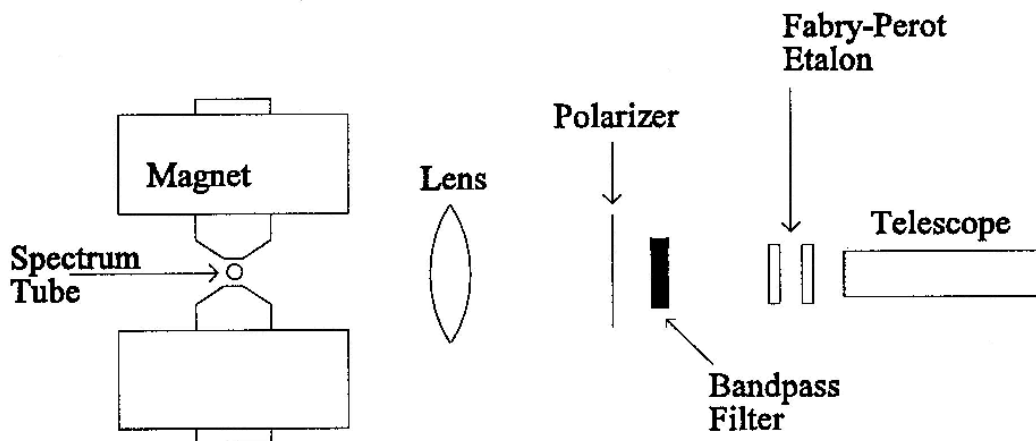


Figure 1: Zeeman effect apparatus (top view).

b) Electronically controlled interferometer

No laser alignment is required in order to see fringes with this interferometer; nevertheless, the mirrors should be aligned for maximum fringe visibility using the electronic tilt controls. Because the plate spacing of this étalon is only $t = 0.5$ mm, the free spectral range is large enough to prevent overlapping of orders even at the highest available magnetic fields.

Using filters for isolation, photograph the fringes of each of the two Hg lines 579.0 nm and 546.1 nm for several different magnetic fields, and with a range of exposure times. (Note that when you observe the 579.0 nm line, there will also be a fringe pattern from a yellow line at 577.0 nm.) It may not be possible to resolve all nine components of the Hg 546.1 nm fringe pattern but a measurement of the separation of the two triplets with $\Delta m_j = \pm 1$ can be used to deduce a value of e/m . Ask for help if you decide to do it this way. After developing and fixing the film, measure the fringe diameters using the scanner on one of the lab computers to produce a large paper copy of the film.

To analyze the data from the electronic interferometer, you will need to understand the concept of fractional order. Review pages 172 – 174 in Ref. 4. Now repeat the derivation of Eq. (4.51) there, but *without* dropping ϵ . You should arrive at the result

$$\theta_p \approx \sqrt{\lambda/t} \sqrt{(p-1) + \epsilon} = \sqrt{2/n_0} \sqrt{(p-1) + \epsilon}.$$

Combining this with Eq. (4.47) we get

$$r_p^2 = f^2 \theta_p^2 \approx \frac{2f^2}{n_0} [(p-1) + \epsilon],$$

which says that as r_p increases quadratically with increasing p . When you look at your photographed rings (pick out the same ring in each order!) you should see this behaviour, and if you plot r_p^2 for a particular ring against p you should find a straight line.

Now why do we care about fractional order ϵ ? Consider two wavelengths $\lambda_1 = 1/\bar{\nu}_1$ and $\lambda_2 = 1/\bar{\nu}_2$ that are *very close* to one another. They have fractional orders ϵ_1 and ϵ_2 at the centre. Now we know that

$$\epsilon_1 = n_0 - n_1 = 2t\bar{\nu}_1 - n_1(1),$$

and correspondingly

$$\epsilon_2 = n_0 - n_1 = 2t\bar{\nu}_2 - n_1(2),$$

where the n_1 's are the (integer) orders of the first rings. Now suppose that the rings do not overlap; then the order $n_1(1)$ of the first ring of λ_1 and the order $n_1(2)$ of the first ring of λ_2 are equal, so subtracting we find

$$\bar{\nu}_1 - \bar{\nu}_2 = \frac{\epsilon_1 - \epsilon_2}{2t}.$$

Thus if we determine the fractional orders for two rings in our Zeeman pattern by using the expression for r_p^2 to fit the positions of the rings, we can deduce the difference in wavenumber between the two rings, and hence the difference in frequency or wavelength. That is, we can use the measured ring spacings to determine the actual Zeeman splitting of the line for a particular field strength B – and thus deduce e/m .

In your report

1. Include your computations of the Zeeman splitting of the levels of the He and Hg spectral lines
2. Show that in a Fabry-Pérot interferometer, overlap occurs when $\bar{\nu}_2 - \bar{\nu}_1 \approx 1/(2t)$
3. Describe your determination of e/m and μ_B from your measurements with the manual interferometer, including an error analysis
4. Describe your determination of e/m from the measurements made with the electronically controlled interferometer

REFERENCES

Zeeman Effect

1. A.C. Melissinos & J. Napolitano, *Experiments in Modern Physics, 2nd Edition*, pp 215 – 228..
2. R. Eisberg and R. Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*, pp 364 – 370.
3. H.G. Kuhn, *Atomic Spectra*.

Fabry-Pérot Interferometer

4. A.C. Melissinos & J. Napolitano, *Experiments in Modern Physics, 2nd Edition*, pp 172 – 177.
5. E. Hecht, *Optics*.
6. Jenkins and White, *Fundamentals of Optics*.