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Abstract: We investigate the understanding of mechanical waves in a class of second-year physics majors at a Canadian university. We administered a previously-developed diagnostic test (Wittmann. Ph.D. thesis, University of Maryland. Unpublished. 1998.) pre- and post-instruction to second-year students, and pre-instruction to a group of first-year students. We find that common misconceptions identified in previous studies involving students in first-year physics courses persist among our second-year students, although the fraction of students holding these misconceptions decreases with instruction. We also find that application of wave concepts becomes more consistent, and that the correlation between the students' own perception of their understanding and their diagnostic test scores increases significantly as their level of instruction advances. We describe two tutorial exercises developed to address areas in which conceptual understanding is weak.

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Résumé : Nous analysons la compréhension des ondes mécaniques chez des étudiants de seconde année dans le programme de physique d'une université canadienne. Nous avons administré un test diagnostique précédemment développé (Wittmann. Thèse de Ph.D., University of Maryland. Non publiée. 1998.) avant et après formation chez des étudiants de seconde année et avant formation chez des étudiants de première année. Nous trouvons que les fréquentes idées préconçues identifiées chez les étudiants de première année persistent chez les étudiants de deuxième année, même si la fraction de ces étudiants qui continuent d'y croire diminue avec le niveau de formation. Nous trouvons aussi que l'application des concepts ondulatoires devient plus cohérente et que la corrélation entre la perception des étudiants de leur propre compréhension et leurs résultats dans notre test s'améliore significativement avec le niveau de formation. Nous décrivons deux tutoriels développés pour sonder les régions du savoir dans lesquels la compréhension des concepts est faible.

[Traduit par la Rédaction]

1. Introduction

The physics of low-amplitude, linear, mechanical waves is routinely taught early in the undergraduate physics curriculum. An understanding of simple waves is fundamental to many more advanced topics in physics, including optics, electromagnetic theory, quantum mechanics, and fluid dynamics. It is, therefore, important for physics educators to appreciate how students think and learn about waves so that the subject can be taught in a way that ensures the development of a correct conceptual understanding that can be carried forward and applied in other areas.

There has been a substantial amount of research on the teaching and learning of introductory physics [1, 2], the bulk of which has focussed on areas such as simple mechanics and electric circuits. Some important general principles have emerged. For instance, it is well known that many students come to class with preconceptions that are at variance with established physics and are difficult to dislodge using traditional lecture-style teaching. Exercises that force students to confront their own predictions with experimental results have proven more effective than lectures at addressing such misconceptions [3]. It is also well-established that students learn

and retain knowledge significantly better when their learning is active rather than passive [2]; this result is not unique to physics, but applies across all disciplines [4].

Despite the importance of mechanical waves, there has been relatively little research on students' learning and understanding of the subject. A 1999 Resource Letter in the American Journal of Physics [1] listed 224 research papers on physics education but only six on waves and sound. Two of these were specifically concerned with sound waves and one with electromagnetic waves. The situation has changed only slightly in the intervening dozen years. A 1992 study used multiple choice questions to investigate French high school and university students' thinking about wave propagation and the mathematical description of waves. A number of conceptual misconceptions were identified, including a common belief that the speed of a wave pulse depends on the pulse's shape and the way in which it is generated [5]. Interviews of (mostly Canadian) university physics graduates who were training to become teachers indicated that many of them had a conceptual understanding of sound propagation that was distressingly wrong [6, 7]. Grayson interviewed students and teachers about kinematics and waves, and incorporated the results into a computer program intended to help students

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understand the motion of a string as a wave propagates along it [8]. In a series of papers, Wittmann and co-workers used interviews and a diagnostic test they developed to investigate students' understanding of wave propagation and superposition [9–12]. In agreement with the earlier work of Maurines [5], they found that many students mistakenly believed that the way a wave pulse is produced (for example, the speed at which a taut string is flicked) will affect the pulse's speed. They also showed that many students held misconceptions about the superposition of wave pulses. Wittmann et al. postulated that students holding these misconceptions were thinking about waves as objects, and proposed a mental model in which ideas from the mechanics of moving particles, such as force, energy, and collisions, were incorrectly and inconsistently applied to waves [9, 12]. Similar models have been discussed by Maurines [5] and by Hrepic et al. [13]. Wittmann et al. also suggested that, in thinking about waves, students combine reasoning resources from many different sources in a way that leads to incomplete and inconsistent conceptual understanding [10]. de Bruyn et al. [14] describe a computer-based first-year course on oscillations and waves that emphasized inquiry-based and collaborative learning. Hrepic et al. investigated the mental models developed by students to understand sound propagation. In addition to the object-based model and the (conventional) wave model, they found evidence for a range of "blended" models that included aspects of both [13]. The way students think about waves at boundaries was investigated by Kryjevskaia et al. [15], who also discussed instructional material designed to address the conceptual difficulties they identified. Podolefsky and co-workers have studied the importance of analogies in learning physics, and applied their model to electromagnetic and sound waves [16, 17] and computer simulations of wave interference [18]. Student understanding of waves in the context of optics and electromagnetic waves has also been studied [19, 20].

As part of their research on student thinking about waves, Wittmann and co-workers developed the University of Maryland Wave Diagnostic Test [9, 21]. It focuses on four areas in which students commonly had conceptual difficulties or incorrect preconceptions: the propagation speed of a wave, superposition, the motion of a suspended particle in a sound wave, and the reflection of wave pulses [9, 12]. Tongchai et al. developed a multiple choice test based on the test of Wittmann et al. which they gave to students in Australia and Thailand [22]. Their findings were consistent with those of Wittmann et al. They performed a statistical analysis of their test results to support its validity as a classroom diagnostic tool [22]. Caleon et al. developed a "three-tier" diagnostic test in which students were asked to rate their confidence in their answers; the intent was to differentiate between answers that are right or wrong because of valid or faulty reasoning and those that are simply guesses [23]. This test was administered to secondary school students in Singapore. All of these diagnostic tests have helped to confirm the existence of the aforementioned misconceptions held by many students.

In the present work, we use the Wave Diagnostic Test of Wittmann et al. to investigate reasoning about waves among second-year physics majors at the University of Western Ontario. The primary purpose of this project was to investigate the extent to which the misconceptions identified in previous studies persist among students who have already had some university-level instruction in wave physics and who, by virtue of their enrollment in a physics program, are presumably both interested in and reasonably capable at physics. Our results from the second-year class are presented and compared with complementary results from a first-year class and with previously published data in Sect. 2. A second goal of this study was to develop teaching materials to correct gaps in student understanding identified from the diagnostic test results. We developed tutorial lessons focused on wave propagation and superposition, which are discussed in Sect. 3. The paper ends with some general discussion in Sect. 4. A brief summary of this work has been submitted for publication elsewhere.1

2. Diagnostic test results

The primary subjects of this study were students in a second-year course on oscillations and waves. This course was new to our undergraduate curriculum, and was taught for the first time by one of us (JdeB) in the fall semester (September to December) of 2010. In addition to providing a unified treatment of the physics of classical oscillations and waves, the course was intended to introduce students to some important theoretical tools that would be used in later physics courses. The course involved three hours of lectures and two hours of "tutorials" per week. The lectures were largely traditional in format, but several in-seat experiments and class demonstrations were used throughout the course. The tutorials were mainly computer-based and held in a computer lab. The students worked on interactive computer-based exercises and homework assignments using Matlab, and discussed course material with peers and instructors. In most tutorials, the students were given online worksheets that provided guided instruction on the numerical investigation of specific problems on oscillations and waves. They were encouraged to discuss these problems with their classmates, teaching assistants, and the course instructor as they worked on them. The tutorial problems were closely linked to the current lecture material. Enrollment was 32 students. Almost all were in their second year of a program in physics, astrophysics, or medical physics, and all had previously had some exposure to mechanical waves in their first-year courses.

We used the long version of Wittmann et al.'s Wave Diagnostic Test [21] to study this group's reasoning about waves. We chose this test because it had been validated and used by others previously [9, 12], so a baseline for comparison with our results already existed. It consists of eight free-response questions and two multiple-choice questions. The students answered the free-response questions, handed them in, and then received the multiple-choice questions. The test was administered to this class twice: pre-instruction in September 2010, and post-instruction in December 2010. We refer to these tests as T_2 (pre) and T_2 (post), respectively. The students were allowed one hour to complete the test, which was given during the tutorial classes. Although participation in the test was optional, 30 of the 32 students wrote the pre-test and 25 wrote the post-test.

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The Wave Diagnostic Test was also administered to students in an enriched calculus-based first-year physics course in the winter semester (January to April) of 2011. This course is intended for students planning to continue their studies in the physical sciences, and oscillations and waves is one of several topics in its curriculum. The test was administered to students from this class pre-instruction during a regularly scheduled lecture period in January 2011; we refer to this test as T_1 . Participation was again optional, and in this case only 15 of the 31 students in the class chose to write the test. This group, therefore, cannot be taken as a representative sample of the class. Because waves was the last topic treated, time constraints prevented the administration of a post-test to this class.

We made one addition to the diagnostic test: The students were asked to indicate their knowledge of waves on a scale from 1, meaning limited knowledge, to 5, representing a solid fundamental understanding of wave concepts. This measure of students' self-perceived level of understanding serves as an indicator of their confidence in their responses, and as such plays the same role on a more global scale as the confidence rating used by Caleon et al. [23].

Scores on the 10 individual questions on the Wave Diagnostic Test are discussed in ref. 24. The distribution of overall scores, S, obtained by each group of students is shown in Fig. 1. The results are summarized in Table 1, which gives the number, N, of students who wrote the test in each case; the mean score, S, and the range of scores; the standard deviation, $\sigma_{\rm S}$, of the distribution; and the standard error, $S_{\rm S}$, of the mean. The mean scores on the the first- and second-year pretests (T_1 and T_2 (pre)) are the same within the standard error. The range is smaller on T_1 than for the other tests, but the number of students who wrote T_1 is also smaller, and, as noted previously, they are likely not a representative crosssection of the class. Comparing Figs. 1b and 1c indicate that the overall distribution of scores (and the mean score, S) in the second-year course shifted higher following instruction, while the width of the distribution remained the same. The improvement in class scores between $T_2(\text{pre})$ and $T_2(\text{post})$ can be quantified using the Hake factor, h [25], defined as

$$h = \frac{S(\text{post}) - S(\text{pre})}{100\% - S(\text{pre})} \tag{1}$$

where S(pre) and S(post) are the mean scores (in %) on the pretest and post-test, respectively, and *h* is the improvement in performance as a fraction of the total possible improvement. For our second-year course, we found h = 0.33. This can be compared to results from a large number of first-year mechanics courses [25] obtained using the Force Concept Inventory [2, 26] or the Mechanics Diagnostic Test [27]; a Hake factor of 0.33 is higher than that of any of the "traditional" lecture-based first-year courses but at the low end of the "interactive engagement" courses included in the data set of ref. 25.

The statistics of the self-perceived understanding score, U, are presented in Table 2. The mean score increases from T_1 through T_2 (pre) to T_2 (post), indicating that the students' confidence in their knowledge of waves increases as they learn more physics, as one would expect. Most interesting, however, is the correlation between the students' perception of their understanding and their actual performance on the diagknow quite well whether or not they understand the subject. Not all of the concepts tested on the Wave Diagnostic Test were explicitly taught in our second-year course. The results from individual questions on the pre- and post-tests were compared to determine the extent to which changes in score depended on whether the topic had been covered in the course lectures or in the tutorials, to be discussed below. Four questions on the test dealt with sound waves, which were not covered in the course at all. Two questions concerned the reflection of a pulse at a wall. Reflection and transmission coefficients at a discontinuity were discussed in the course lectures, but the specific case treated on the test - a pulse reflecting from a fixed or free end of a string was not discussed explicitly. This topic was also not treated in the tutorials. Finally, another four questions concerned the propagation of a wave pulse on a string and superposition, topics that were covered in both the lectures and the tutorials. The Hake factors calculated for each of these three groups of questions are presented in Table 3. The class average improved for all groups of questions, even the group on topics that were not part of the course material. The Hake factor was lowest and had the highest variability ($h = 0.16 \pm 0.27$, mean \pm standard deviation) for the "no instruction" questions. Scores on one question in this group, involving the dependence on volume and pitch of the time taken for a shout to travel between two people, actually decreased significantly. On the other hand, the question for which the improvement was largest was also in this group; it concerned the motion of a suspended dust particle due to a passing sound wave. We found the same average level of improvement for the other two groups of questions: $h = 0.30 \pm 0.08$ for the questions on material covered in the lectures and $h = 0.30 \pm 0.06$ for questions on material covered in both lectures and tutorials. As the standard deviations indicate, the improvements on the individual questions in these two groups were substantially more uniform than in the "no instruction" group.

These results suggest, perhaps reassuringly, that any sort of instruction is better than none. Both the tutorials and the lectures appear to have had a positive impact on student understanding of waves. On the other hand, some improvement was still observed on the "no instruction" group of questions. We interpret this to indicate that a more comprehensive understanding of the wave concepts taught in the course also improved understanding of related concepts that were not explicitly taught.

Three pairs of questions on the Wave Diagnostic Test concern similar concepts, and in fact have similar wording. One pair concerns the superposition of wave pulses; another involves the motion of a suspended particle in front of a speaker, and the third involves the reflection of a wave pulse at a boundary. Correlations between the scores on each question in a pair were analyzed to determine the extent to which students applied the relevant concepts consistently [22]. TaFig. 1. Overall scores on the Wave Diagnostic Test for students on the (a) first-year pre-test, T_1 ; (b) second-year pre-test, T_2 (pre); and (c) second-year post-test, T_2 (post).



Table 1. Summary of the overall scoresS (%) from the three writings of theWave Diagnostic Test.

	T_1	$T_2(\text{pre})$	$T_2(\text{post})$
Ν	15	30	25
S _{min} (%)	28	3	19
S _{max} (%)	60	71	97
S (%)	40	43	62
$\sigma_{\rm S}$ (%)	11	17	17
$S_{\rm S}~(\%)$	3	3	3
h			0.33

Note: Symbols are defined in the text.

Table 2. Students' self-perceived under-standing scores, U, from the three tests.

	T_1	$T_2(\text{pre})$	$T_2(\text{post})$
U	2.53	2.93	3.96
$\sigma_{ m U}$	0.64	0.94	0.62
$S_{\rm U}$	0.17	0.17	0.03
r	-0.01	0.34	0.81

Note: U, mean self-perceived understanding score; σ_U , standard deviation; S_U , standard error; and r, correlation coefficient between U and S, the scores on the diagnostic test.

ble 4 shows the correlation coefficient, r, for these pairs of questions on the three tests. Note that while a value of r close to one would indicate strong consistency, it could just as well mean "consistently wrong" as "consistently right."

Table 3. The Hake factor, *h*, for groups of test questions sorted by how the material was covered in the second-year course.

Instruction method	h
No instruction	0.16±0.27
Lectures	0.30 <u>+</u> 0.08
Lectures and tutorials	0.30 ± 0.06

Table 4. Correlation coefficients, *r*, for pairs of test questions on similar material.

Торіс	T_1	$T_2(\text{pre})$	$T_2(\text{post})$
Superposition	-0.14	0.55	0.55
Particle in a sound wave	0.09	0.40	0.47
Reflection	-0.16	-0.09	0.18

The results from the first-year class show very small and, in two of the three cases, negative — correlations between the scores on the two questions in a pair. This indicates that the responses of these students are essentially random: there is no significant level of consistency in their responses. This is no doubt because these students had not had any instruction in waves beyond high school. The second-year students displayed a reasonably high correlation coefficient ($r \approx 0.5$) on the questions on superposition (which, as noted earlier, was explicitly covered in the course) and the motion of a particle in a sound wave (which was not), indicating consistent application of concepts in these two cases. Interestingly, though, the level of consistency did not change from the pre-test to the post-test. In contrast, the correlation coefficients for questions on the reflection of a wave pulse were quite low, although r increased slightly from $T_2(\text{pre})$ to $T_2(\text{post})$. This likely reflects the fact that the specific example treated on the test was not covered in the second-year course.

The specific responses to the test questions were analyzed to identify commonly held misconceptions. Our findings are consistent with previous work [5, 9, 22] and so will not be discussed in detail, but we present a summary here for completeness. Many students (60% on T_1 , 30% on T_2 (pre), and 28% on $T_2(\text{post})$ gave responses indicating a belief that the speed of sound depends on frequency. It is clear from the context of the course and from the detailed responses on the diagnostic test that this belief does not stem from a deep understanding of dispersion, but rather, as noted by others [9, 12], from a misapplication of the equation $c = \lambda f$, where c is the speed of sound, λ the wavelength, and f the frequency. Rather than recognizing that c is a material property and that changing f will result in a corresponding change in λ , with c remaining constant, some students reason that an increase in f implies an increase in c. Particularly on T_1 , many students indicated incorrect reasoning about the motion of a suspended particle in a sound wave: only 20% of the first year students responded correctly to a multiple-choice question on this topic, while 80% gave answers consistent with the sound being a transverse wave or producing a net force in the direction of propagation. The fraction of correct responses on this question increased to 37% on T_2 (pre) and 65% on T_2 (post). On the free-response version of this question (i.e., on which there was no prompting based on the choices available), the conceptual misunderstanding among the first-year students was even more manifest: no students responded correctly on T_1 . The fraction of correct responses was 40% on T_2 (pre) and 72% on T_2 (post), similar to the results on the multiple choice version of the question. Another common misconception was identified from responses to a question on the speed of a wave pulse on a string. As in the particle pulse model of Wittmann et al. discussed earlier [9], many students believe that moving one's hand up and down faster or harder will increase the propagation speed of the pulse. We found that 93% of the responses on T_1 indicated this type of thinking, with the fraction decreasing to 83% on $T_2(\text{pre})$ and 32% on $T_2(\text{post})$. We also found students to have substantial conceptual difficulties with superposition, as in previous work [9]. On T_1 , 40% of the students were unable to correctly answer a question on the superposition of two pulses of opposite sign; the results from $T_2(\text{pre})$ and $T_2(\text{post})$ were very similar. On a question involving the superposition of two positive pulses, 87% of the responses on T_1 were incorrect, with the fraction of incorrect responses decreasing to 62% on $T_2(\text{post})$. Most of the incorrect responses indicated that students were adding the two waves together only at the peaks of the pulses, and not at all points of the waveform.

3. Tutorials

Based on the results of the diagnostic test and on previous work [5, 9, 22], we developed two tutorial exercises specifically intended to address misconceptions held by a significant fraction of the students. One of these concerned the propagation speed of a wave pulse; the other, superposition of waves. As discussed earlier, a common misconception is that changing the way in which a pulse is produced will change its speed. In general, the test results indicated that students had a shaky understanding of what factors actually determine the speed of a wave on a string: few recognized that the tension plays a role, and even fewer that the mass density of the string would affect the speed, despite the fact that this material had been covered in their first-year physics courses. We designed an interactive experiment on pulse propagation that was performed by the students in about forty-five minutes during one of the weekly tutorial periods. The exercise followed the predict-confront-resolve strategy developed by McDermott [3] and involved measuring the speed of pulses produced by hitting a stretched spring in a number of different ways. Before doing the experiments, the individual students were asked to predict whether the pulse speed in each scenario would increase, decrease, or stay the same relative to a baseline measurement. The students then formed groups of three, discussed their individual predictions, and came up with group predictions for each case. The groups then performed the experiments. Two of the students held the ends of the spring (a "Slinky") a prescribed distance apart. One of them, the "wavemaker," hit the spring near its end with his or her hand to produce a transverse pulse. The third student used a stopwatch to measure the time for the pulse to travel back and forth along the spring three times. The results of five trials were averaged and the speed of the pulse determined. After these baseline measurements were made, the groups were asked to hit the spring "harder," to displace the spring further in the same amount of time (in other words, to create a pulse with the same width but a larger amplitude), and to increase the tension in the spring by stretching it more. In each case, the students made measurements of the speed of the wave pulses averaged over five trials.

The predictions made by the individual students are shown in Table 5. Notwithstanding the diagnostic test results, most students individually predicted the correct result for all three scenarios. A significant number of students, however, thought that hitting the spring harder would cause the speed to increase. Some students wrote down the reasoning behind this prediction, saying that hitting it harder would cause the frequency to increase, and thus cause the speed to increase. Some students predicted that displacing the string further in the same amount of time would cause the speed to decrease; when given, the reasoning was that the wavelength would increase in this scenario, causing the speed to decrease. Interestingly, this reasoning is inconsistent with the reasoning discussed earlier that is used by some students to justify an increase in speed with frequency. Discussion among peers led to correct group predictions in all but one case, as shown in Table 5.

The experimental results are also shown in Table 5. All groups obtained the expected result in scenario 3, in which the tension in the spring was increased by stretching it further, and in fact all groups measured a substantial increase in the speed compared to the baseline. Eight of the 10 groups found the expected results in scenarios 1 and 2. Even though the average speed differed slightly between scenarios, the students were astute enough to recognize that these changes could be accounted for by the experimental error inherent in the experiment. Two groups, however, measured increases in

Table 5. Predictions and result from the tutorial on wave speed.

Speed will	Increase	Decrease	Not change	
Individual predictions	mereuse	Deereuse	enunge	
individual predictions			10	
Scenario 1: hit harder	9	0	19	
Scenario 2: greater amplitude	0	4	24	
Scenario 3: increase tension	25	2	1	
Group predictions				
Scenario 1: hit harder	0	0	9	
Scenario 2: greater amplitude	0	1	8	
Scenario 3: increase tension	9	0	0	
Experimental results				
Scenario 1: hit harder	2	0	8	
Scenario 2: greater amplitude	2	0	8	
Scenario 3: increase tension	10	0	0	

the propagation speed that were too large to be considered within experimental error. These results are likely due to unfortunate experimental technique: when hitting the spring "harder," the wavemaker could simultaneously pull back on the spring, increasing the tension.

This exercise involved the students actively moving around and doing the experiments themselves, either in the computer lab in which the tutorial was held or in the hallway outside the lab. They were observed to ask each other questions about the activity and to discuss it with their peers. The fact that the groups were able to come up with correct predictions in almost all cases, even if some members had made incorrect individual predictions, implies that the group discussion was effective in persuading students with incorrect ideas to change their minds.

The other tutorial exercise dealt with superposition, which is another area in which the test results indicated conceptual difficulties. We developed a two-hour, two-part, Matlabbased tutorial through which the students could simulate and experiment with the superposition of waves. This exercise was carried out near the end of the course, shortly before the post-test. It built on Matlab programs and techniques the students had developed earlier in the course and applied them to the solution of a new problem, and in a sense served as a culminating activity for the tutorials. In a previous tutorial class, the students had written a Matlab script to animate a traveling wave pulse of the form $f(x \pm ct)$, where x is the position and t the time. An online worksheet, in the same format used for other tutorial classes in the course, led them through the process of modifying their traveling-pulse program to simulate, and to plot an animation of, two counterpropagating pulses and their superposition. The second part of the tutorial involved calculating and plotting Fourier sums, another example of superposition that had been discussed in the course lectures. The students wrote a program to calculate an adjustable number of coefficients in a Fourier series, then add the appropriate sine waves together and display the results graphically. They were asked to look at both triangular waves and square waves and to investigate how the number of terms in the Fourier sum changed the shape of the resultant wave. The students worked on this exercise independently, although, as usual for tutorial classes in this course, there was a substantial amount of discussion among students and with the teaching assistants throughout the tutorial.

Apart from some minor programming issues, the first part of this tutorial went smoothly. The students enjoyed experimenting with different pulse shapes and visualizing the effects of changing parameters. The second part of the tutorial gave some of the students a little more trouble, both with the programming and with the physics. This was likely a result of them being asked to do too much in a single tutorial period, a situation that will be rectified in future versions of the exercise. The animations they produced allowed them to visualize immediately the results of the quantitative problem they were solving, and in writing and modifying their programs, they were forced to become involved in their own learning and to take an active role in applying the concepts they had learned in the lectures.

4. Discussion

Data from a single class clearly constitute a limited data set. Nonetheless, our results provide some interesting information on the reasoning of second-year physics students about mechanical waves. Our diagnostic test results indicate that many of our students hold the same misconceptions about waves held by students at first-year and lower levels [5, 9, 22]. This is the case despite the fact that they received instruction on the subject in first year, and despite the fact that, on average, one would expect this group of students to be more interested in, and to have a better conceptual understanding of, elementary physics than an average student in first-year physics. It is reassuring that the test scores increased and the fraction of students holding these alternative conceptions decreased between the second-year pre-test and the post-test, but it is disturbing that this fraction was still around 30% even following second-year instruction (by, it must be said, an experienced instructor who was aware of the problem beforehand). This indicates the extreme persistence of these misconceptions.

Our test results, obtained with second-year students at a Canadian university, are quite consistent with previous results found with students from France [5], the United States [9], Thailand and Australia [22], and Singapore [23]. This clearly indicates that the common alternative conceptions are not the result of any particular cultural factors, textbooks, or educational systems. Rather, they seem to be universal. It would thus be extremely interesting to investigate their origin.

Some misconceptions stem in part from a difficulty in consistently applying mathematical reasoning to physical systems [5]. The flawed application of the equation $c = \lambda f$ discussed earlier is one example. We also found that some students were quite capable of writing down the equation for a propagating sound wave, for example, but were unable to apply it correctly to determine what happens to a particle suspended in such a wave. A correct and consistent conceptual understanding is vital for a thorough understanding of waves (or any other topic in physics). An understanding of the connection between the abstract mathematical description of a phenomenon and the physical reality is equally important, particularly as one progresses to more advanced areas and applications of physics.

We found that performance improved on questions that were the subject of instruction through lectures and (or) tutorial exercises as well as on questions dealing with topics that

were not explicitly discussed in the course. Improvement was stronger for the topics that were covered, but the fact that scores improved in the other areas as well suggests that the students experienced a general increase in conceptual understanding of all aspects of wave physics over the course. Quite apart from the improvement in scores, the increased correlation between the students' self-assessment of their understanding and their actual test scores from T_1 through T_2 (post) also suggests an improvement in understanding and maturity of thought as they progress through the program; they become better able to assess their own state of knowledge. We also found that the correlations between results on related test questions increased between first and second year, indicating that the more advanced students reasoned more consistently. Interestingly, however, there was no significant change in these correlations between $T_2(\text{pre})$ and $T_2(\text{post})$.

5. Conclusion

We have used the Wave Diagnostic Test [12] to investigate the reasoning of first- and second-year physics students about mechanical waves. The performance of the second-year class improved significantly from pre-test to post-test, with a Hake factor of 0.33. The correlation between test scores and the students' self-perceived level of understanding also increased with instruction. Our results indicate that many of our second-year students hold the same misconceptions about waves as students at other levels and from other countries. A correlation analysis showed that the first-year students did not apply wave concepts consistently. The second-year students appeared to apply concepts reasonably consistently on questions about superposition and the motion of a particle in a sound wave, but not on questions about the reflection of a pulse. Interactive tutorial exercises were developed on the propagation of a wave pulse and superposition, two topics with which students had conceptual difficulties. These exercises were useful in helping students to develop an improved understanding of the subject.

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