Physics 2110A
Fall 2010

Assignment 1 – Due Monday, September 27, 2010

1. **A problem with numbers:** Problem 1.1 from the text.

2. **Linearity of SHM:** In class we derived the ordinary differential equation for simple harmonic motion,

$$\frac{d^2x}{dt^2} = -\omega^2 x\,.$$

We showed that the form $A\cos\omega t$ was a solution of this equation. I then told you that the most general solution had the form

$$x(t) = A\cos\omega t + B\sin\omega t.$$

Verify by direct substitution that this form of $x(t)$ solves our differential equation. This is an example of the *principle of linear superposition*, which you saw in several contexts in first year. It works because $\frac{d(x+y)}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$, and because our equation depends only linearly on x and its derivatives. Later on we will study equations of motion that are *nonlinear*, in which case superposition does not work and life becomes both more complicated and more interesting!

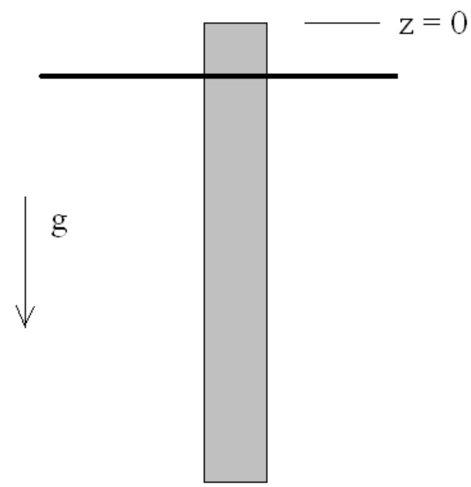
3. **A bobbing log:** A vertical log of mass m is partly submerged in the waters of Lake Ontario, as shown in the diagram. The cross sectional area of the log is A, its submerged length at equilibrium is ξ, and its density is ρ. The density of the water is ρ_w. The buoyant force on the log is equal to the weight of the water displaced by the log. We define $z = 0$ to be the position of the top of the log when the log is at equilibrium.

   a. Determine the buoyant force on the log at its equilibrium position, and argue that at equilibrium, this force exactly balances the force on the log due to gravity.

   b. Now imagine displacing the log downwards by a distance z. Determine the forces on the log, and use Newton's second law to derive a differential equation for the position of the top of the log as a function of time. You should end up with an equation analogous to that we derived in class for a mass on a spring, meaning that the log will bob up and down in simple harmonic motion.

   c. What is the angular frequency of the log's oscillation?

   d. Show by calculating the work done in moving the log from its equilibrium position to position z that the potential energy of the system is $\frac{1}{2}A\rho_w g z^2$. (See Eq. (1.18) in the text for a hint.) Using your result, give an expression for the total energy of the system, and show that it has the form of Eq. (1.43c) in the text.



e. Based on your knowledge of the properties of logs, use reasonable values for the relevant parameters to come up with a numerical value of the oscillation frequency. Does your result seem ok?

4. **Fitting data:** The Excel file posn.xls contains position vs. time data obtained from an experiment on an oscillating mass on a spring (just like the one we studied in the tutorial class). It also contains three cells in which various constants are defined.
    a. Use Excel to plot a graph of position vs. time, and label it appropriately. Just use symbols, with no connecting lines.
    b. Set up your spreadsheet to calculate the function $A\cos(\omega t+\varphi)$ for all of the values of t in the original data set. Use the constants defined in cells B2, B3, and B4 of the spreadsheet as the parameters A, ω, and φ respectively. Plot your cos function on the same graph as the data, using only a line, no symbols. Your function won't look much like the data at first, because the parameter values are not correct. Now adjust the values of A, ω, and φ by changing the contents of cells B2, B3, and B4. The graph will update itself automatically. Find the parameter values that give the best agreement between your cos function and the original data. Print out and submit your graph along with your best-fit parameter values.

       What you have done here is fit a function to the data, allowing you to determine the values of the parameters that best describe the data. Fortunately there are better and easier ways of doing this than adjusting the parameters by hand! The most commonly used technique is called least-squares fitting – ask Dr. Simpson about it next term.

5. **Two-dimensional SHM:** A particle undergoes simple harmonic motion in both the x and y directions simultaneously. Its x and y coordinates are given by

$$x = a\sin\omega t$$
$$y = b\cos\omega t\,.$$

    a. Eliminate t from these equations to show that the path of the particle in the x-y plane is an ellipse.
    b. Calculate the kinetic and potential energy at a point on the particle's trajectory. Show that the ellipse is a path of constant total energy, and show that the total energy is given by the sum of the separate energies of the x and y oscillations.
    c. Show that the quantity $x\dot{y}-y\dot{x}$ is also constant along the ellipse. What is the physical meaning of this quantity?