SECTION A: NUCLEAR AND PARTICLE PHENOMENOLOGY

This introductory section covers some standard notation and definitions, and includes a brief survey of nuclear and particle properties along with the major steps from a historical perspective. Then the properties of "stable" nuclei are described.

A.1 Introductory survey of nuclear and particle properties

Nucleus ➔ system of interacting neutrons (zero charge) + protons (positive charge)
The interactions are important, because they (together with the substructure of the neutron and proton, are ultimately responsible for all the other different particles.
Rest mass of neutron (m_n) ≈ Rest mass of proton (m_p) >> Rest mass of electron (m_e)
Approximately, $m_n = 1839\, m_e$ and $m_p = 1836\, m_e$ where $m_e = 9.109 \times 10^{-31}$ kg $= 0.511$ MeV/c^2.

We denote

A = mass number = no. of protons + no. of neutrons (= no. of nucleons)
Z = no. of protons
$N = A - Z$ = no. of neutrons.

A specific nucleus corresponding to an atom of element X is denoted as ${}^{A}X^{Z}$ or more compactly as (A, Z).

Some terminology is

Isotope – same Z (different A and N)
e.g., ${}^{35}Cl^{17}$ and ${}^{37}Cl^{17}$ are isotopes of Cl
Isotone – same N (different A and Z)
e.g., ${}^{27}Al^{13}$ and ${}^{28}Si^{14}$ are isotones
Isobar – same A (different Z and N)
e.g., ${}^{36}S^{16}$ and ${}^{36}A^{18}$ are isobars

Just as an atom can either be in its ground state or a higher-energy excited state (due to the electron states), so can a nucleus be excited from its ground state to excited states, which are sometimes referred to as *resonances* or *isomers*.

Historical landmarks in nuclear and particle physics

1) *Discovery of the electron* (J. J. Thomson 1897). He discovered electrons as negatively charged particles (charge $-e$) emitted from heated metal cathodes. The particles were given speed v by accelerating them through a potential difference V. Equating loss of PE to gain in KE gives (classically)

$$\frac{1}{2} m_e v^2 = eV$$

The electrons were then passed through a combination of perpendicular magnetic B and electric E fields acting as a velocity selector giving $v = E/B$. This enabled the charge-to-mass ratio to be found:

$$\frac{e}{m_e} = \frac{v^2}{2V} = \frac{1}{2V}\left(\frac{E}{B}\right)^2$$

Thomson obtained the same numerical value (irrespective of the cathode material used), indicating the electrons were a fundamental particle.
About 15 years later, Millikan carried out his famous oil-drop experiment which gave a value for e (and hence for m_e). Approximately, $e = 1.602 \times 10^{-19}$ C.

2) *Existence of the nucleus* (Rutherford ~ 1910–1911). Rutherford carried out a series of experiments where he projected positively-charged alpha particles (now known to be He nuclei) at heavier atoms. By analysing the angles of scattering, he was able to deduce that the atom has a dense core containing most of the mass.

3) *Discovery of the proton* (Rutherford 1919). In similar scattering experiments with nitrogen as the target material, Rutherford was able to produce a disintegration process and hydrogen nuclei were released as one of the products. These had a positive charge, equal in magnitude to that of the electron, but were much more massive. They were called protons (p).

4) *Discovery of the neutron* (Chadwick 1932). Chadwick bombarded targets of light elements (such as Be or B) with high-energy alpha particles. It was observed that reactions took place in which an uncharged particle with mass roughly the same as a proton were emitted. These were called neutrons (n) and their discovery explained where the additional mass of the nucleus came from.

5) *Discovery of the positron* (Anderson 1932). Anderson was studying certain types of radioactive decay processes, and he detected (in a cloud chamber experiment) the path of a new particle. From its behaviour in E and B fields, it was deduced to have the same mass as the electron but the opposite charge $+e$. It was called the positron (e^+ to distinguish it from the electron e^-) and had actually been predicted earlier by Dirac's theory of relativistic QM. The positron is an example of an *antiparticle*, i.e., a particle with the same mass but with other properties (like the charge) having the opposite sign. It was inferred from Dirac's theory that there should also be an antiparticle to the proton.

6) *Discovery of the neutrino and antineutrino* (existence deduced ~1930s, but first detected in 1956 by Reines and Cowan). They are antiparticles of one another and occur in β decay which occurs in two forms:

$$\beta^- \text{ decay:} \quad (A,Z) \rightarrow (A,Z+1)+e^- + \bar{\nu} \quad (\bar{\nu} = \text{antineutrino}) \quad \text{or} \quad n \rightarrow p+e^- +\bar{\nu}$$

$$\beta^+ \text{ decay:} \quad (A,Z) \rightarrow (A,Z-1)+e^+ + \nu \quad (\nu = \text{neutrino}) \quad \text{or} \quad p \rightarrow n+e^+ +\nu$$

Notice that Z changes by +1 or -1, so the nucleus changes to a different chemical element (because it is energetically favourable - see later), while A remains unchanged.

The neutrino and antineutrino have no charge, and for a long time it was believed that they have a rest mass equal to zero - for which special relativity implies that they travel at or near the speed of light c. They do, however, carry both energy and momentum, and this is how their existence was first established.

A specific example of β decay is

$$^{77}\text{As}^{33} \rightarrow {}^{77}\text{Se}^{34} + e^- + \bar{\nu}$$

7) *Discovery of mesons*. Originally this term was applied, starting in the 1930s, to particles with intermediate mass (between that of the electron and the neutron or proton). They were first predicted theoretically in 1935 by Yukawa by analysing forces between nucleons, and the first mesons to be detected around 1947 were the unstable charged pi mesons (or pions) with mass ~ 250-300 m_e.

Subsequently other intermediate mass particles were found and labelled as mesons (some misleadingly like the mu meson, now known as a muon).

Nowadays the term "meson" is applied more specifically (i.e., with respect to the strong nuclear interaction and the quark model) - see later.

8) *Advances in high-energy particle accelerators* (starting ~ 1950s and 1960s, and continuing to the present). Large accelerators to produce high-energy beams (GeV and more) of particles and colliding beams of particles came into operation and gave rise to very many (~100 or so) new particles, most of which were highly unstable

and decayed into other particles. Examples of accelerators are the Stanford linear accelerator (SLAC) and the circular collider at CERN (in Switzerland).

9) *Theory of quarks* (Gell-Mann and Zweig and others, mid-1960s and 1970s). The theory was initially based on the idea that there were so many particles that they were unlikely to be all elementary (i.e., they could instead be classified more neatly in terms of "building blocks" of other particles). These became known as "quarks" and it was necessary for them to have a charge that is a fraction of the basic electronic charge e. Also experimental evidence from particle accelerators soon became available to show the proton has an internal structure (with charge and mass concentrated in three lumps), as predicted.

To go further we need the classification of all *forces of interaction* in physics as follows:

TYPE	*RELATIVE STRENGTH*	*RANGE*
Strong	1	Short range ($\sim 10^{-15}$ m)
Electromagnetic	1 / 137	Long range ($1 / r^2$)
Weak	$1 / 10^9$	Short range ($\sim 10^{-18}$ m)
Gravitational	$1 / 10^{38}$	Long range ($1 / r^2$)

The name *hadron* is given to particles that can take part in the strong interaction (either alone or more typically together with other types of interaction). There are lots of particles in this category, and it is usually subdivided into baryons (such as n or p) and mesons (such as the pion π).
The quark model applies to hadrons only, with 3 quarks making up a baryon and 2 quarks (or antiquarks) making up a meson (see later).

10) *The standard model* (~ 1980-ish to present). This is the currently accepted model in particle physics: it is a particle theory that combines the strong, electromagnetic and weak interactions. It includes hadrons (via the quark theory) along with other particles (called leptons and gauge bosons) - see later.
The existence of one of the last "missing pieces" in the standard model, namely the Higgs boson (anticipated in the 1960s), was finally confirmed in 2012.

<u>Other related properties</u>

Statistics and spin
All particles can be categorized as either *fermions* or *bosons*, depending on whether their intrinsic spin angular momentum (or spin) quantum number is an (integer + ½) or an integer.

Fermions obey Fermi-Dirac statistics and are subject to the Pauli Exclusion Principle in QM. Examples are e^-, e^+, ν, n and p.

Bosons obey Bose-Einstein statistics and the Pauli Exclusion Principle does not apply. Examples are mesons, such as π, and gauge bosons, such as photons and W particles.

The net spin of a nucleus with its Z protons and N neutrons is deduced by combining the individual spins of its constituents using the rules of QM (see later).

Magnetic dipole moments
The spin angular momentum of a particle generally gives rise to an associated magnetic dipole moment. For an electron the magnetic moment is

$$\boldsymbol{\mu} = g \frac{e}{2m_e c} \mathbf{S}$$

where **S** is the spin angular momentum vector and the constant g = 2 is called the Landé factor. Since the quantized values of the spin in any direction are ±½ $\hbar$, it follows that the magnitude of the magnetic moment is

$$\frac{e\hbar}{2m_e c}$$

This is called the Bohr magneton μ_B and is equal to 5.89 × 10^{-11} MeV/T.

The magnetic moments of nucleons (neutrons or protons) are given by similar expressions, except that $g \neq 2$ and the electron mass is replaced by the corresponding nucleon mass.
By convention, the unit for nuclear magnetic moments is the nuclear magneton μ_N defined by

$$\mu_N = \frac{e\hbar}{2m_p c}$$

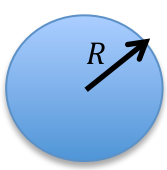
It is smaller than the Bohr magneton by a factor of m_p/m_e or ~2000. The intrinsic magnetic moments of the proton and neutron are measured to be

$$\mu_p \approx 2.79\,\mu_N \quad \text{and} \quad \mu_n \approx -1.91\,\mu_N$$

Conservation laws and symmetries
In nuclear and particle physics (just as in other areas of physics) there are certain conservation laws, typically associated with some basic symmetry of the system.
Familiar examples are conservation of total energy (or mass-energy), linear momentum and angular momentum. Also total net charge is always conserved.

Other symmetries are of interest in particle physics (and often lead to the introduction of new quantum numbers - see later).
For example, until the 1950s it was believed that parity conservation was a symmetry property of all fundamental interactions (in other words, that the physical processes would be the same in a right-handed set of axes as in a left-handed set of axes). Then it was discovered that, although parity is a conserved quantity if only strong and electromagnetic interactions are involved, it is not necessarily conserved with weak interactions. This will be looked at in more detail later, along with other examples of symmetries for particles.

Size and shape of nuclei
Experiments (e.g., using high-energy scattering of protons) show that most nuclei are approximately spherical in shape. Also they have a fairly definite boundary with radius R given roughly by



$$R = r_0 A^{1/3}$$

where r_0 is an empirical constant with dimensions of length.
It is found that $r_0 \sim 1.2$ fm [1 fm = 10^{-15} m = 1 fermi]

Therefore typically nuclei radii ~ 10^{-15} m to 10^{-14} m (compared with atomic radii ~ 10^{-10} m)
It follows that

$$\text{Density of nuclear matter} = \text{mass / volume} \propto A/(A^{1/3})^3 \propto \text{constant}$$

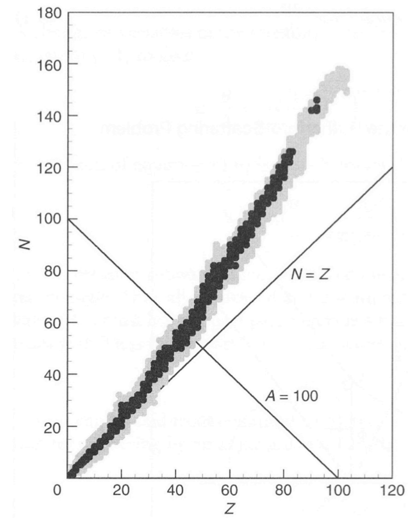
The property of definite boundary and constant density gives a broad analogy with liquid drops.
Distortions from spherical shape can be detected through asymmetries in the observed charge distribution (of the protons), giving rise to quadrupole moments depending on the spin axis.

A.2 General properties of stable nuclei

Stability rules for nuclei

Observations on stable nuclei have led to empirical stability rules (in terms of the A, N and Z values):

1. Typically $N \sim Z$ for small nuclei (with Z less than about 20 or 30), but for larger nuclei there is an increasing tendency for $N > Z$ in stable nuclei.



2. Both N and Z show a tendency to be even integers, rather than odd integers. The evidence is based on:

a) Abundance in the Earth's crust:

Estimates give ~ 90% associated with even Z, e.g., Ca^{30}, Si^{14}, O^{8}

b) Isotopic abundance:

Taking Mg^{12} as an example, the abundances are roughly

$^{24}Mg^{12}$ 80% ; $^{25}Mg^{12}$ 10% ; $^{26}Mg^{12}$ 10%

c) Number of stable isotopes:

Nuclei with Z even tend to have many isotopes (e.g., $^{16}O^{8}$, $^{17}O^{8}$ and $^{18}O^{8}$ for O^{8}; 10 isotopes of Sn^{50})

d) Only four or five stable nuclei with both N and Z odd:

These are $^{2}H^{1}$, $^{6}Li^{3}$, $^{10}B^{5}$, $^{14}N^{7}$, plus (maybe) $^{18}F^{9}$

e) Abundance of isobars:

These have same A as in $^{124}Sn^{50}$ and $^{124}Te^{52}$.

For A even, there are ~ 40 pairs of isobars

For A odd, there are ~ 3 pairs of isobars (e.g., $^{113}In^{49}$ and $^{113}Cd^{48}$).

Measurement of nuclear mass

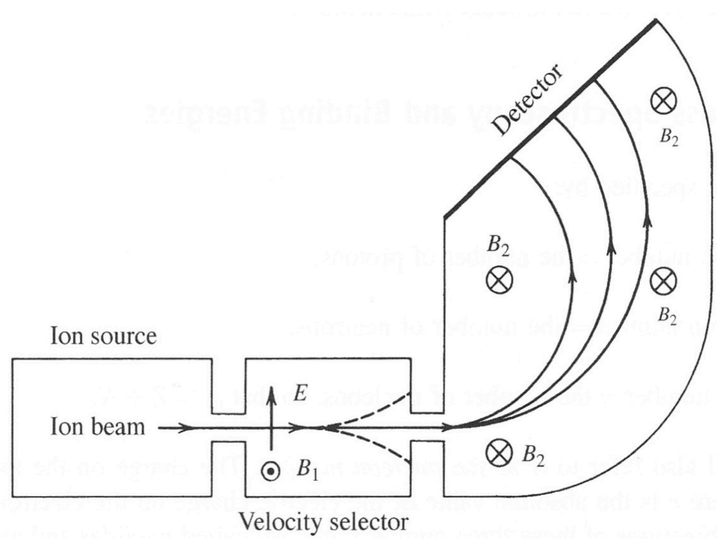
Briefly, some of the experimental methods for nuclei are:

1. Use of mass spectrometer

Typically, a collimated beam of ions is produced, which is then passed through a velocity selector (so the beam is mono-energetic), and the ions are deflected by an arrangement of E and/or B fields.

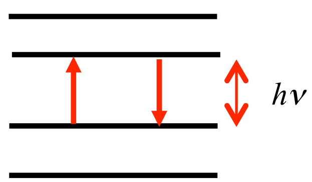
➔ From the deflection, deduce q/m (where q is the charge on the ion) and hence deduce m.

Finally, allowance has to me made for the electrons in the ion to deduce the nuclear mass.



2. Microwave spectroscopy

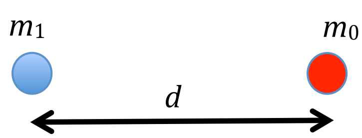
This applies to rotations of simple molecules about their centre of mass. There will be energy levels corresponding to the quantized rotational states of a molecule:



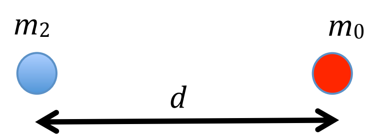
Here $h\nu$ = change in rotational energy, where ν is frequency of the em radiation (typically in the microwave or GHz region).
Now rotational energy = $(\text{angular momentum})^2 / 2I$
where I is the moment of inertia, and the angular momentum will be quantized in QM.

So the procedure is: Measure ν ➔ deduce I ➔ deduce masses (if the bond lengths are known).

For example, consider two diatomic molecules corresponding to different isotopes (masses m_1 and m_2) of one of the nuclei (and denote the other nuclear mass by m_0):



and



Now $\nu \propto 1/I$, and so the ratio of frequencies is

$$\frac{\nu_2}{\nu_1} = \frac{I_1}{I_2} = \left(\frac{m_0 m_1}{m_0 + m_1}\right) d^2 \div \left(\frac{m_0 m_2}{m_0 + m_2}\right) d^2 = \frac{m_1(m_0 + m_2)}{m_2(m_0 + m_1)}$$

Measure ν_2 / ν_1. Then if m_0 is known, we can deduce the ratio between the masses m_1 and m_2.

This method has been applied to $^{12}\text{C} - ^{16}\text{O}$ and $^{13}\text{C} - ^{16}\text{O}$, giving the ratio of the ^{13}C mass to the ^{12}C mass as about 1.083613.

3. Cyclotron resonance

A charged particle in a constant, uniform magnetic field B will move in a circle in the plane perpendicular to the applied field - this is known as cyclotron motion. The Lorentz force of magnitude $q\,B\,u$, where u is the speed of the particle gives the central force, so classically

$$qBu = \frac{mu^2}{r}$$ where r is the radius of the orbit.

Then, if ν is the frequency for 1 orbit (= cyclotron frequency),

$$\text{Time for 1 cycle} = \frac{1}{\nu} = \frac{u}{2\pi r} = \frac{B}{2\pi}\left(\frac{q}{m}\right)$$

Measure ν and B to deduce q/m ➔ m if q is known.

4. Energy balance in nuclear reactions

Simple nuclear reactions can sometimes be used to deduce unknown masses, provided some of the other masses are already known.

For example, consider $\quad {}^7\text{Li}^3 + {}^1\text{H}^1 \rightarrow {}^4\text{He}^2 + {}^4\text{He}^2$

From conservation of energy, $\quad M({}^7\text{Li}^3)\,c^2 + m_p\,c^2 \rightarrow 2M({}^4\text{He}^2)\,c^2 + Q$

Measure the energy release Q (from the KE of the He nuclei and the incident proton).
If two of the above masses are known, we can deduce the third mass.

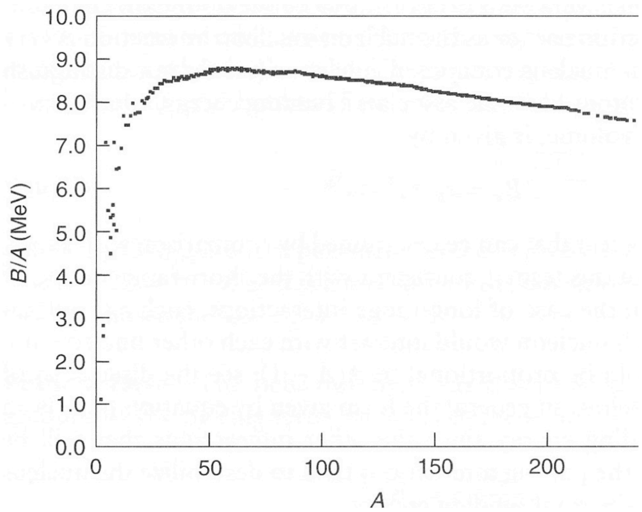
A.3 Binding energy of stable nuclei

To summarize, a stable nucleus is made up of N neutrons and Z protons, each having rest mass m_n and m_p respectively. However, the measured mass M is always slightly less than the sum of the individual masses, with the difference representing the total energy that holds the nucleus together. We denote

$$Mc^2 = (Nm_n + Zm_p)c^2 - B$$

where the extra term $-B$ is the binding energy (B.E.).
If $B > 0$, the nucleus is stable. If not, it is unstable.
Typically, B is of order 1% of Mc^2 and it generally increases with $A = N + Z$, roughly in proportion to A.

It is generally more useful to look at the B.E. per nucleon, as given by the ratio B/A.



The main conclusion is that (except for the smallest nuclei): $B/A \sim$ constant (at about 8 MeV).
This behavior is often referred to as *saturation* of the nuclear forces.

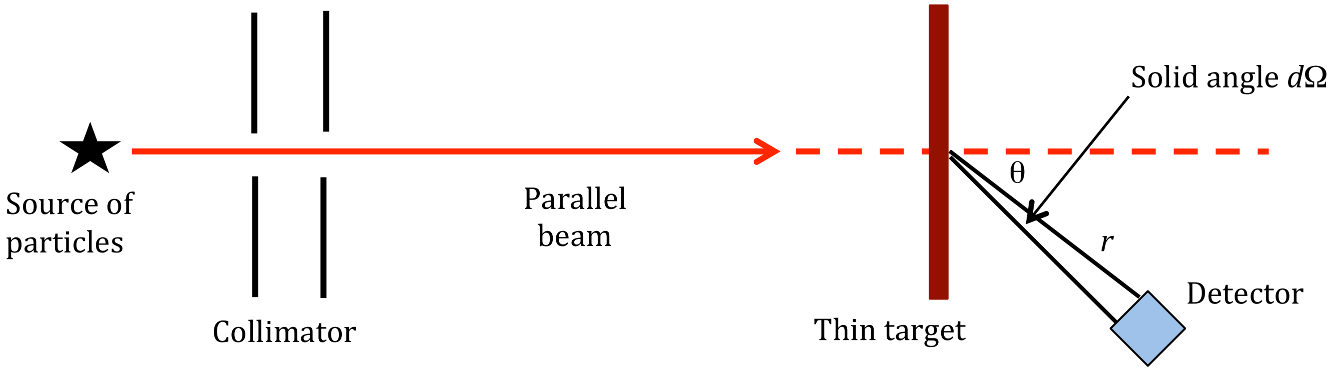
The saturation effect is contrary to the expected behavior for "classical forces":
Total PE of binding $\approx E_0 \times$ Number of interacting pairs of nucleons (where E_0 is a constant)
$\approx E_0 \times \frac{1}{2} A(A-1) \approx E_0 \times \frac{1}{2} A^2$ for large A
Therefore we conclude that $B/A \approx \frac{1}{2} E_0 A \propto A$, which is incorrect.
Later we show that saturation of nuclear forces is a QM effect.

Two additional topics will be covered separately:

Appendix A1 – Scattering of Particles

Appendix A2 – Particle Accelerators

APPENDIX A1: SCATTERING OF PARTICLES

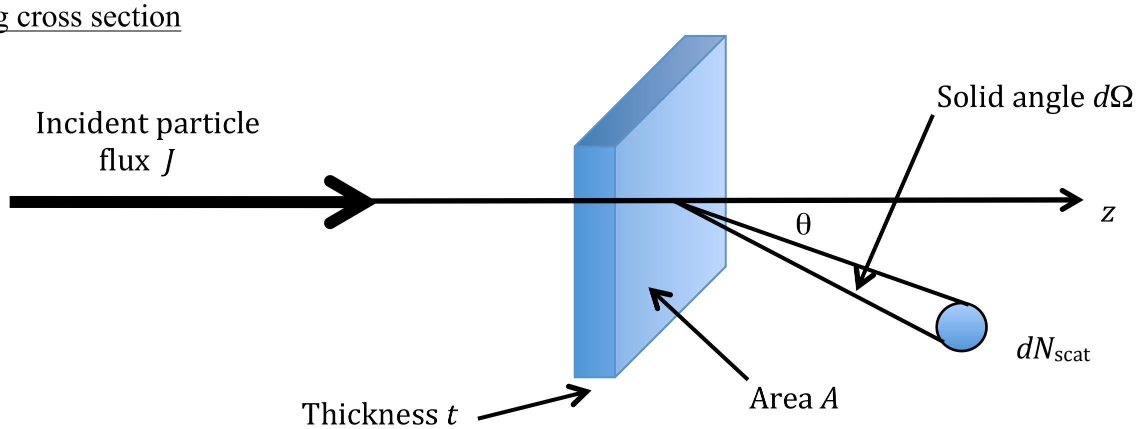


The source is of monoenergetic particles (e.g., p, n, α, β, etc). Typically the target is chosen to be sufficiently thin so that each particle undergoes no more than one collision.
The area (denoted by dS) at the detector) will typically be small enough that the solid angle $d\Omega$ is very small:

$$d\Omega = \frac{dS}{r^2} \qquad \text{for the elementary solid angle}$$

The measurement of the scattering process is generally interpreted in terms of a *scattering cross section*.

Scattering cross section



Suppose J is the incident particle flux (i.e., number of particles per unit area per unit time) and dN_{scat} is the number of particles scattered per unit time into elementary solid angle $d\Omega$.
Let n = number of scattering centres per unit volume in the target, so total number of scattering centres = $A\,t\,n$

We expect dN_{scat} to be proportional to $A\,t\,n$ and J and $d\Omega$

Therefore we write $dN_{scat} = I(\theta, \phi)\ A\,t\,n\ J\,d\Omega$

where $I(\theta, \phi)$ is the proportionality factor (which may depend in general on the usual polar angles θ and ϕ, although the ϕ dependence can often be ignored).

$I(\theta, \phi)$ has the dimensions of area and is known as the *differential scattering cross section*.

The *total scattering cross section* (denoted by σ) is defined by

$$N_{scat} = \sigma\ A\,t\,n\ J$$

where N_{scat} is the total number of particles scattered per unit time in all directions.

It follows that the connection between the two cross sections is

$$\sigma = \int I(\theta,\phi)\, d\Omega$$

Differentiating, we can also write $\dfrac{d\sigma}{d\Omega} = I$ which provides us with an alternative notation.

Note that in spherical polar coordinates,

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

so

$$\sigma = \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi\, I(\theta,\phi)$$

Both σ and I have the dimension of area, so the natural SI unit is m^2. This is not very practical in nuclear and particle because it is much too large. A practical-size unit in this case is the barn (b) defined by

$$1 \text{ b} = 10^{-28} \text{ m}^2$$

An important example of scattering in nuclear physics is the so-called *Rutherford scattering* of particles with positive charge *ze* by target nuclei with (positive) charge *Ze*, typically at nonrelativistic energies. The interaction is via the Coulomb repulsion with

$$V(r) = \frac{zZe^2}{4\pi\varepsilon_0 r}$$

It can be shown (by classical mechanics for the trajectory) that the differential scattering cross section is

$$I(\theta,\phi) = \left(\frac{zZe^2}{16\pi\varepsilon_0 T}\right)^2 \frac{1}{\sin^4(\theta/2)} \qquad (T = \text{incident particle KE})$$

with no dependence on angle ϕ.

Quantum mechanical theory of scattering

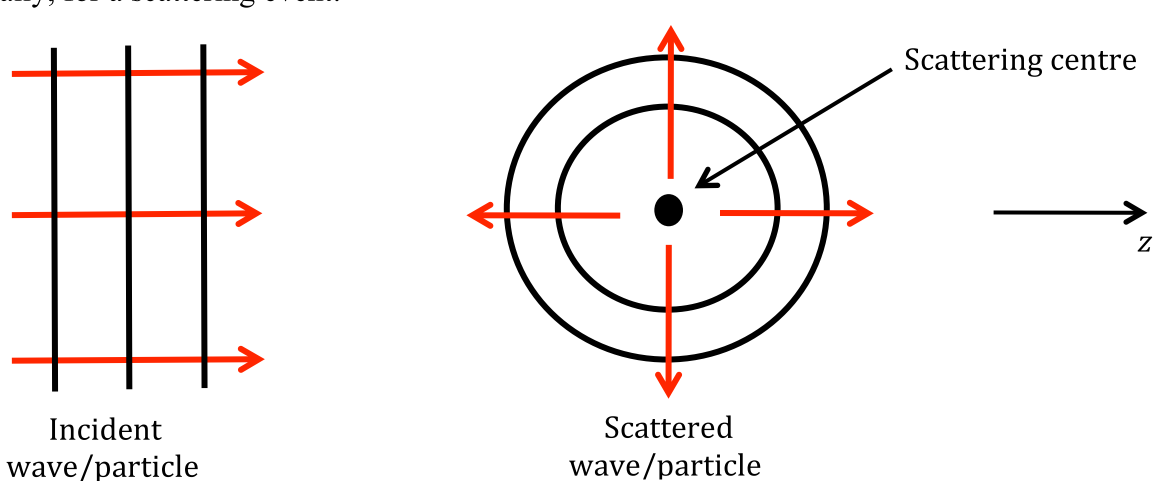
The method involves using Schrodinger's equation (so it is a nonrelativistic treatment), together with the principle of superposition, to form the wave function $\psi(\mathbf{r})$.

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right]\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Here we are considering particles that have rest mass m and energy E being scattered by an interaction potential $V(r)$, taken to be spherically symmetric for simplicity. After rearranging, this can be written as

$$\left[\nabla^2 + k^2 - U\right]\psi = 0 \qquad \text{where} \qquad k^2 = \frac{2mE}{\hbar^2} \quad ; \quad U = \frac{2mV(r)}{\hbar^2}$$

Schematically, for a scattering event:-



We use spherical polar coordinates (r, θ, ϕ) with the origin at the scattering centre and the polar z axis along the propagation direction of the incident wave/particle.

We can consider the asymptotic solutions at large distances r, where we have $V(r) \approx 0$ since the interaction falls off with r. Then the total wave function will be a superposition of eigenfunctions of

$$\left[\nabla^2 + k^2\right]\psi = 0$$

that represent the incident plane wave and spherical scattered wave.
So, for large r we expect

$$\psi \approx e^{ikz} + \frac{e^{ikr}}{r} f(\theta)$$

where the first term is a plane wave traveling in the z direction (representing the incident particle) and the second term is a spherical wave traveling radially outwards (representing a scattered particle).
The angular dependence of the scattering is described by the function $f(\theta)$, assuming for simplicity no ϕ dependence.

Relation between scattering cross section and $f(\theta)$

A standard result from QM is that the flow of particles can be described in terms of the *particle current* **J** which is defined in terms of the wave function by

$$\mathbf{J} = -\frac{i\hbar}{2m}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right)$$

So, incident particle current is $J_z = -\frac{i\hbar}{2m}\left(\psi_I^* \frac{d}{dz}\psi_I - \psi_I \frac{d}{dz}\psi_I^*\right)$ with $\psi_I = e^{ikz}$

This yields $J_z = \frac{\hbar k}{m}$, as expected (it looks like momentum divided by mass)

For the particle current scattered radially, we use the expression for radial component of ∇ in polar coordinates.

So, $J_r = -\frac{i\hbar}{2m}\left(\psi_S^* \frac{d}{dr}\psi_S - \psi_S \frac{d}{dr}\psi_S^*\right)$ with $\psi_S = \frac{e^{ikr}}{r} f(\theta)$

If we keep only the leading order terms as $r \to \infty$, this gives approximately

$$J_r = \frac{\hbar k}{mr^2}|f(\theta)|^2$$

Now we can relate this to the scattering cross section:

The number of particles per unit time scattered across an element of area dS at the detector is

$$dN_S = J_r\, dS = \frac{\hbar k}{m}|f(\theta)|^2 \frac{dS}{r^2}$$

$$= J_z |f(\theta)|^2\, d\Omega$$

But by definition the differential scattering cross section is simply

$$I(\theta) = \frac{dN_S}{J_z d\Omega}$$

Therefore we get the simple relation that

$$I(\theta) = |f(\theta|^2$$

This result provides the basis of QM scattering theory: if we can solve for $f(\theta)$ by solving Schrodinger's equation for wave function ψ from first principles, then we have the solution to the scattering problem. Examples of this will come later.

It is obvious that $f(\theta)$ must have the dimensions of length. It is sometimes called the *scattering length*.

One important approach is called the *method of partial waves*. It involves making an expansion of $f(\theta)$ in terms of Legendre polynomials $P_\ell(\cos\theta)$ for values $\ell = 0, 1, 2, \ldots$, of the orbital angular momentum quantum number. They are polynomials of degree ℓ with $\cos\theta$ as the variable. For example, the first few Legendre polynomials are

$$P_0(\cos\theta) = 1\,, \quad P_1(\cos\theta) = \cos\theta\,, \quad P_2(\cos\theta) = (3\cos^2\theta - 1)/2\,, \quad P_3(\cos\theta) = (5\cos^3\theta - 3\cos\theta)/2$$

The same polynomials show up in other areas of physics (such as in electromagnetism and in solving Schrodinger's equation for the electronic states of the H-atom). The expansion for $f(\theta)$ in the present case using Legendre polynomials is mathematically rather similar to making the so-called "multipole expansions" for the scalar and vector potentials used in electromagnetism.

In many cases of practical importance for nuclear and particle physics only the terms with low value of ℓ are important, and this gives a simplification. The condition is roughly that

$$\ell \quad < \quad r_0 k$$

where r_0 is the 'range' of the interaction and we recall that k is related to the energy E by

$$k \quad = \quad \sqrt{2mE}/\hbar$$

Therefore if the range is small (e.g., less than about 1 fm in the case of the strong interaction, and much smaller in the case of the weak interaction), this limits the range of ℓ values needed.

In simple cases, only $\ell = 0$ will be appreciable $\Rightarrow$ S wave scattering (or just S scattering).
If $\ell = 1$ $\Rightarrow$ P scattering.
If $\ell = 2$ $\Rightarrow$ D scattering, etc.

In some cases there might not be a well-defined range r_0 (for example, in long-range interactions such as the Coulomb interaction potential which goes like $1/r$). In these cases the method of partial waves is not particularly useful, so other methods must be used to find $f(\theta)$.

Examples of different kinds of scattering will come up later in the course.