

Final Examination

Name: _____

Time allowed: 3 hours

The use of simple calculators (with no stored information) is permitted. A **formula sheet** (using the same notations as during the classes) is attached; no other aids are permitted.

There are *two sections*:

Section A consists of longer questions and is worth 64% of the total marks; you should answer **any four** (out of the five) questions in this section.

Section B consists of shorter questions and is worth 36% of the total marks; you should answer **all** of the questions in this section.

Use the space on this exam paper to provide your answers. The marking scheme (out of 100) is indicated.

SECTION A

Answer **any four** of these questions

A.1 [16 marks]

A.2 [16 marks]

A.3 [16 marks]

A.4 [16 marks]

A.5 [16 marks]

SECTION B

Answer **all** of these questions

B.1 [9 marks]

B.2 [9 marks]

B.3 [9 marks]

B.4 [9 marks]

Formula Sheet (for Final)Sections 2 and 3

$$\begin{aligned}
\mathbf{F} &= \frac{1}{4\pi\epsilon_0} \frac{qQ}{r_e^2} \hat{\mathbf{r}}_e & \mathbf{E}(\mathbf{r}) &= \frac{1}{4\pi\epsilon_0} \int_{\text{volume}} \frac{\rho(\mathbf{r}')}{r_e^2} \hat{\mathbf{r}}_e d\tau' & \oint_{\text{surface}} \mathbf{E} \cdot d\mathbf{a} &= \frac{1}{\epsilon_0} Q_{\text{enc}} \\
\nabla \cdot \mathbf{E} &= \frac{1}{\epsilon_0} \rho & \mathbf{E} &= -\nabla V & \nabla^2 V &= -\frac{\rho}{\epsilon_0} & V(\mathbf{r}) &= \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{r_e} d\tau' & C &= \frac{Q}{V} \\
\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} &= \frac{1}{\epsilon_0} \sigma \hat{\mathbf{n}} & W &= \frac{1}{2} \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \sum_{j=1(i \neq j)}^n \frac{q_i q_j}{r_{eij}} & W &= \frac{1}{2} \sum_{i=1}^n q_i V(\mathbf{r}_i) & \nabla \times \mathbf{E} &= 0 & \mathbf{p} &= q\mathbf{d} \\
W &= \frac{\epsilon_0}{2} \int_{\text{allspace}} E^2 d\tau & \mathbf{f} &= \frac{1}{2\epsilon_0} \sigma^2 \hat{\mathbf{n}} & P &= \frac{\epsilon_0}{2} E^2 & W &= \frac{1}{2} CV^2 & W &= \frac{1}{2} \int \rho V d\tau \\
V(\mathbf{r}) &= \frac{1}{4\pi\epsilon_0} \left[\frac{1}{r} \int \rho(\mathbf{r}') d\tau' + \frac{1}{r^2} \int r' \cos\theta' \rho(\mathbf{r}') d\tau' + \frac{1}{r^3} \int (r')^2 \left(\frac{3}{2} \cos^2\theta' - \frac{1}{2} \right) \rho(\mathbf{r}') d\tau' + \dots \right] & V_{\text{dip}}(\mathbf{r}) &= \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}
\end{aligned}$$

Sections 4 and 5

$$\begin{aligned}
\mathbf{p} &= \alpha \mathbf{E} & \mathbf{N} &= \mathbf{p} \times \mathbf{E} & \mathbf{F} &= \mathbf{p} \cdot \nabla \mathbf{E} & \sigma_b &= \mathbf{P} \cdot \hat{\mathbf{n}} & \rho_b &= -\nabla \cdot \mathbf{P} & \mathbf{D} &= \epsilon_0 \mathbf{E} + \mathbf{P} \\
\nabla \cdot \mathbf{D} &= \rho_f & \oint \mathbf{D} \cdot d\mathbf{a} &= Q_{f-\text{enc}} & \mathbf{P} &= \epsilon_0 \chi_e \mathbf{E} & \mathbf{D} &= \epsilon \mathbf{E} & \epsilon &= \epsilon_r \epsilon_0 & \epsilon_r &= 1 + \chi_e \\
V(\mathbf{r}) &= \frac{1}{4\pi\epsilon_0} \int_V \frac{\hat{\mathbf{r}}_c \cdot \mathbf{P}(\mathbf{r}')}{r_c^2} d\tau' & \mathbf{F} &= Q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})] & \mathbf{F}_{\text{mag}} &= \int I(d\mathbf{l} \times \mathbf{B}) & \nabla \cdot \mathbf{J} &= -\frac{\partial \rho}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0 & \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} & \oint \mathbf{B} \cdot d\mathbf{l} &= \mu_0 I_{\text{enc}} & \mathbf{B}(\mathbf{r}) &= \frac{\mu_0}{4\pi} \int \frac{d\mathbf{l}' \times \hat{\mathbf{r}}_c}{r_c^2} & \mathbf{B} &= \nabla \times \mathbf{A} \\
\mathbf{A}(\mathbf{r}) &= \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{r_c} d\tau' & \nabla \cdot \mathbf{A} &= 0 & \nabla^2 \mathbf{A} &= -\mu_0 \mathbf{J} & \mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} &= \mu_0 (\mathbf{K} \times \hat{\mathbf{n}}) & \mathbf{A}_{\text{dip}}(\mathbf{r}) &= \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2} \\
\mathbf{A}(\mathbf{r}) &= \frac{\mu_0 I}{4\pi} \left[\frac{1}{r^2} \oint r' \cos\theta' d\mathbf{l}' + \frac{1}{r^3} \oint (r')^2 \left(\frac{3}{2} \cos^2\theta' - \frac{1}{2} \right) d\mathbf{l}' + \dots \right] & \mathbf{m} &= I \int d\mathbf{a} = I\mathbf{a}
\end{aligned}$$

Sections 6 and 7

$$\begin{aligned}
\mathbf{N} &= \mathbf{m} \times \mathbf{B} & \mathbf{F} &= \nabla(\mathbf{m} \cdot \mathbf{B}) & \mathbf{J}_b &= \nabla \times \mathbf{M} & \mathbf{K}_b &= \mathbf{M} \times \hat{\mathbf{n}} & \mathbf{B} &= \mu_0 (\mathbf{H} + \mathbf{M}) & \nabla \times \mathbf{H} &= \mathbf{J}_f \\
\mathbf{M} &= \chi_m \mathbf{H} & \mathbf{B} &= \mu \mathbf{H} & \mu &= \mu_0 (1 + \chi_m) & \mathbf{J} &= \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}) & R &= \frac{L}{\sigma A} & \sigma &= \frac{nfq^2 \lambda}{2mv_{\text{thermal}}} \\
P &= IV = I^2 R & \Phi &= \int \mathbf{B} \cdot d\mathbf{a} & \mathcal{E} &= -\frac{d\Phi}{dt} & \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \Phi_2 &= M_{21} I_1 \\
M_{21} &= \frac{\mu_0}{4\pi} \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r_c} & \Phi &= LI & W &= \frac{1}{2} LI^2 & W_{\text{mag}} &= \frac{1}{2} \int (\mathbf{A} \cdot \mathbf{J}) d\tau = \frac{1}{2\mu_0} \int B^2 d\tau \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} & \mathbf{J}_d &= \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} & \mathbf{J}_p &= \frac{\partial \mathbf{P}}{\partial t} \\
\text{Maxwell's equations:} & \nabla \cdot \mathbf{D} = \rho_f & \nabla \cdot \mathbf{B} &= 0 & \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \nabla \times \mathbf{H} &= \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}
\end{aligned}$$