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The Continuum Re-processing Efficiency of Be Star Circumstellar Disks

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ABSTRACT
We have calculated the total flux emitted in Hα, Pα, and Brα by the circumstellar envelope of both an early Be star, γ Cas, and a late Be star, 1 Delphini, assuming the central star is the only source of energy input into the circumstellar envelope. These estimates are based on the Be star models of Millar & Marlborough (1998, 1999a&b) which have self-consistent temperature distributions determined by equating the local rates of energy gain and energy loss in the envelopes. We find that an additional source of ionizing photons, as argued by Apparao (1998), is not necessary to account for the observed emission.

Key words: stars: emission-line, Be-stars: early-type-(stars:) circumstellar matter

1 INTRODUCTION

In 1866 Vatican astronomer Angelo Secchi visually identified Hβ emission in the star γ Cas (Secchi 1867). This detection marked the first observation of the class of stars now known as Be. These are main sequence or slightly evolved stars which show, or have shown, emission in the Balmer lines. The origin of this emission is principally due to radiative recombination in a disk-like envelope of ionized circumstellar gas. Be stars can exhibit variations in emission on a wide range of time scales. The mechanism which creates and maintains these disks is not fully understood. Rapid rotation certainly contributes, but because rotation is typically only ~ 0.8 of the critical velocity, another mechanism must be functioning. Proposed models to explain the formation of these disks include, for example, the wind compressed disk model (Bjorkman & Cassinelli 1993), non-radial pulsations (Vogt & Penrod 1983), and non-spherical, radiatively-driven wind models (Puls, Petrenz & Owocki 1999).

The disk-like circumstellar gas is highly concentrated in the equatorial regions surrounding the star (Waters 1986). Compared to a typical H II region, the disk-like envelope has larger densities, smaller geometrical dilution of stellar radiation, significant hydrogen photoionization from excited states, and supersonic macroscopic velocities. Typical values of temperature, outflow velocity, and number density in the envelope are 10^4 K, 100-200 km s^{-1}, and 10^{12}cm^{-3}, respectively. Hence, many of the simplifying assumptions which apply to H II regions are invalid. As a consequence, ad hoc models have been developed for Be star envelopes by necessity, due to the complexity of the equations which describe the structure and dynamics of the circumstellar material. The Poeckert Marlborough model (1978), hereafter PM, is one such model which has been successful in describing some aspects of these disks for a range of Be stars. In the PM model an exponential density distribution perpendicular to the equatorial plane is assumed, and as a result, the gas is dense in and near the equatorial plane but decreases rapidly in density with increasing distance from the plane. Recent direct images of several Be stars (Dougherty & Taylor 1992, Quirrenbach et al. 1997) and spectropolarimetric results (Wood et al. 1997) demonstrate that Be star disks are indeed quite thin making the PM model attractive.

For most models of the past, it has been customary to assume either a constant temperature for the entire envelope or a simple temperature distribution with the temperature decreasing as a power law with increasing distance from the central star. In contrast to this, Millar & Marlborough (1998), hereafter MM, developed a method to determine the disk temperature self-consistently, thereby eliminating the need to assume an envelope temperature distribution. MM modified the PM code to calculate the energy gain and loss rates at positions in the gas. If, by chance, the correct temperature were assumed at a particular position, the rate of energy gain there would equal the rate of energy loss. If not, the temperature was adjusted iteratively and atomic level populations re-calculated until the energy gain and loss rates were equal. The result of this procedure is a self-consistent set of temperatures for various positions throughout the envelope. See MM and Millar & Marlborough (1999b) for further details as applied to γ Cas, an early-type Be star (B0 IVe), and Millar & Marlborough (1999a & b), for 1 Del, a late-type Be star (B8-9e). This work demonstrates that it

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is possible to reproduce relative line strengths, namely Hα, with a self-consistent temperature distribution.

However, Apparao & Tarafdar, hereafter AT, in a series of papers (1987, 1997a&b), have argued that there is not sufficient stellar UV continuum radiation present in late type Be stars to ionize enough gas in order to produce the observed absolute fluxes. Recently, Apparao (1998) has reiterated the results of their work. We, along with Apparao (1998), note that there is in principle more than enough flux in the stellar continuum to account for the observed line emission; the debate is over re-processing efficiency. AT claim that the efficiency of Be star disks in converting stellar continuum photons into emission lines is simply not high enough for later spectral types. We disagree with this conclusion.

In this paper, we evaluate the total flux escaping from our model Be envelopes, in contrast to that along a particular line of sight, in the important Hα, Pα, and Brα transitions. To do so, we estimated a local escape probability for line photons in each transition from each volume element in the circumstellar disk. This simple treatment is consistent with the inclusion of radiative line losses in the self-consistent temperature disk models of MM, and should be of sufficient accuracy to address the large deficiencies ($\sim 10^3$) claimed by AT for the late-type Be stars. This work represents the first time that the flux calculations have been obtained with models which have self-consistent temperatures determined throughout the circumstellar envelope. Others have calculated these fluxes, but in each case the envelope temperatures were assumed (Kastner & Mazzali 1989, Stee et al.1995). Before presenting these flux estimates we will first discuss the role played by line optical depths.

2 CALCULATIONS: Hα, Pα, AND BRα LINE FLUXES

2.1 Pure Case B Recombination

In their analysis, AT assume that the observed emission lines are formed by pure recombination, under case B conditions, in the ionized disk of the Be star. If this assumption is correct, the total flux escaping in the $j \to i$ transition of hydrogen is given by

$$F\,(ji) = h\nu_{ji}\,\alpha_{ji}^{B} \int_{\rm Disk} {N_e}^2\, dV = h\nu_{ji}\,\alpha_{ji}^{B}\, EM_{\rm Disk}\,, \qquad (1)$$

where h is Planck's constant, ν_{ji} is the line frequency, α_{ji}^{B} is the appropriate case B recombination coefficient, N_e is the number density of free electrons, and the integration is over the volume, which we, unlike AT, take to be a disk. Thus the line flux is proportional to the emission measure of the disk, $EM_{\rm Disk}$. The required atomic data and recombination coefficients for the three hydrogen transitions considered in this work (Hα, Pα, and Brα) are given in Table 1. The recombination coefficients adopted are for Te = 10^4 K and log(Ne) = 6, the same conditions as adopted by AT, in order to compare directly to their computed fluxes.

In this approximation, the Hα flux, for example, is controlled only by $EM_{\rm Disk}$ and thus an estimate of N$_e$ is required throughout the disk. AT include photoionization from hydrogen levels n= 1 and 2 and also approximately account for the thermalization of Lyα, as the latter is well known to significantly increase the n= 2 population, making Balmer photoionizations more important (for example see Kwan & Krolik 1981 in the case of AGN). AT assume a model with a shell geometry, 10^{12} cm from the central star, and consider only radially outwardly and inwardly propagating rays. AT conclude that case B recombination fluxes do not reproduce the maximum Hα fluxes of Ashok et al.(1984) for Be stars later than $\sim$ B5. However, their calculation is performed at only one position in the envelope having an assumed constant density, and they also *assume* a constant gas temperature of 10^4 K.

Table 1. Atomic data and recombination coefficients.

Transition	j	i	λ μm	A$_{ji}$ s^{-1}	α_{ji}^{B} cm^{3}s^{-1}
Hα	3	2	0.6563	4.470e+7	1.16e-13
Pα	4	3	1.8751	8.986e+6	3.75e-14
Brα	5	4	4.0512	2.699e+6	3.94e-14

The α_{ji}^{B} recombination coefficients are for Te = 10^4 K and N$_e$ = 10^6cm^{-3} and are adopted from Osterbrock (1989).

In order to compare with AT as directly as possible, we have computed the hydrogen line fluxes using equation (1) for our models of γ Cas and 1 Del. This calculation includes all relevant atomic processes for a 5 level hydrogen atom, enforces radiative equilibrium, and uses the 2D disk geometry of PM. Fluxes from two models for each Be star are presented, one constructed with, and one without, the on-the-spot approximation for the diffuse radiation generated within the envelope (Osterbrock 1989). The diffuse radiation increases the degree of ionization due to increased photoionization from level n = 1. With the diffuse radiation included, the density required at the stellar surface to match the observed relative strength of Hα is approximately a factor of 3 lower for both γ Cas and 1 Del (Millar & Marlborough 1999b). Note that the density at the stellar surface fixes, by continuity, the density throughout the equatorial plane.

Figure 1 shows the derived temperatures for γ Cas using the parameters of PM and MM. Due to the exponential density distribution perpendicular to the equatorial plane and to the high equatorial densities, there are significant temperature variations both near the star and near the equatorial plane. These temperature variations lead to ionization gradients within the circumstellar envelope which may prove to be a valuable probe into the structure of these envelopes.

The hydrogen line fluxes of equation (1) are presented in column 3 of Table 2 for γ Cas and 1 Del, both of which have self-consistently determined envelope temperatures. In addition, unlike AT, a realistic disk geometry has been used. For comparison, typical observed values of the Hα luminosity range from approximately 10^{34}erg s^{-1} for early type Be stars to 10^{32}erg s^{-1} for late type Be stars (Ashok et al, 1984).

In the following sections, we shall demonstrate that many of the above assumptions are incorrect and that the emission line fluxes cannot be reliably computed with an expression such as equation (1). In addition to using the correct geometry of the emitting region (a disk), optical depth effects and other line formation mechanisms, such as collisional excitation, must be included. We now address these remaining important physical effects.

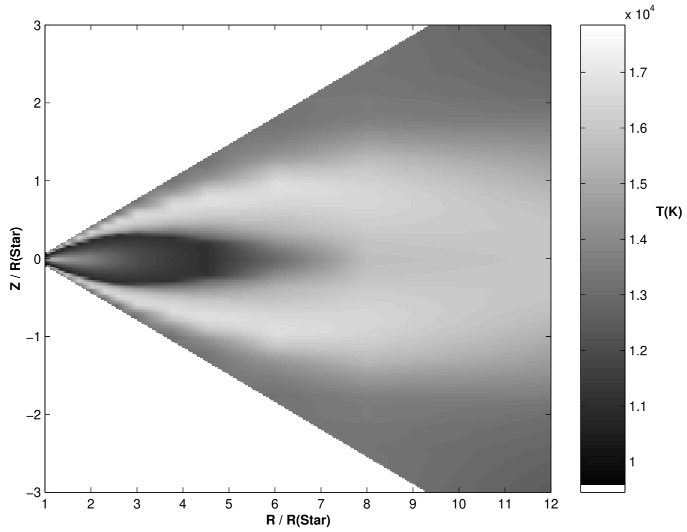

Figure 1. The temperature for which the local energy gain balances energy loss for the circumstellar disk of γ Cas as a function of distance from the rotation axis (R) and height above the equatorial plane (Z).

Table 2. Calculated hydrogen line fluxes.

Star	Diffuse Radiation	Eq.(1) [erg s^{-1}]	Eq.(2) [erg s^{-1}]	Eq.(3) [erg s^{-1}]
			Hα Flux	
γ Cas	yes	8.3x10^{35}	2.1x10^{32}	5.9x10^{33}
	no	2.3x10^{36}	6.3x10^{31}	6.9x10^{33}
1 Del	yes	2.4x10^{34}	7.5x10^{28}	1.3x10^{32}
	no	3.1x10^{33}	4.9x10^{29}	2.3x10^{32}
			Pα Flux	
γ Cas	yes	8.1x10^{34}	1.5x10^{32}	2.5x10^{32}
	no	2.4x10^{35}	1.4x10^{32}	3.1x10^{32}
1 Del	yes	2.7x10^{33}	3.8x10^{31}	2.7x10^{31}
	no	3.5x10^{32}	6.6x10^{30}	5.8x10^{30}
			Brα Flux	
γ Cas	yes	4.1x10^{34}	1.4x10^{32}	1.4x10^{31}
	no	1.2x10^{35}	2.0x10^{32}	2.3x10^{31}
1 Del	yes	1.3x10^{33}	3.7x10^{31}	3.4x10^{30}
	no	1.7x10^{32}	1.1x10^{31}	1.3x10^{30}

γ Cas is B0IVe star with a luminosity of 1.3x10^{38} erg s^{-1} and a radius of 10 solar radii. 1Del is a B8-9e shell star with a luminosity of 9.5x10^{35} erg s^{-1} and a radius of 3.3 solar radii.

2.2 Inclusion of Line Optical Depths

It is assumed in equation (1) and by AT that all photons generated by recombination escape. We have found, however, that for our models there are portions of the envelope both near the star and the equatorial plane that are optically thick in some or all of Hα, Pα, and Brα. Figure 2 shows the line centre optical depth from the equatorial plane to the upper edge of the envelope for each hydrogen transition. Very large optical depths are predicted, particularly close to the star where the densities (and Ne – see figure insert) are large. From Figure 2 it is clear that the line fluxes cannot be given by equation (1) as not all the photons can escape.

As an ad hoc correction for this effect, we have computed the fluxes using

$$F\,(ji) = h\nu_{ji}\,\alpha^B_{ji}\int_{\mathrm{Disk}} P^{esc}_{ji}\,{N_e}^2\,dV\,, \tag{2}$$

where P^{esc}_{ji} is the fraction of photons which escape from the volume element. P^{esc}_{ji} is estimated based on the line center optical depth, τ_o, along a path perpendicular to the equatorial plane from the volume element to the upper edge of the envelope. See MM or Marlborough (1969) for additional details; P^{esc}_{ji} corresponds to the cases to which these papers refer. Typically, for large τ_o, $P^{esc} \propto \tau_o^{-1}$ and thus the optical depth corrections to the escaping flux are very large. Column 4 of Table 1 contains results based on equation (2). Large changes in the hydrogen line fluxes are apparent with reductions in the Hα fluxes to levels far below observation. Two conclusions follow: (i) it is imperative to account for the optical depths and (ii) case B recombination is not an accurate approximation.

2.3 Escape Probability Fluxes

Equation (2) is still not correct, however, as the hydrogen ionization balance is not correctly given by case B recombination due to the optical depth effects previously mentioned. A proper estimate of the hydrogen line fluxes includes both collisional excitation and optical depth effects, and can be obtained by using the standard escape probability approximation for the flux divergence,

$$F\,(ji) = h\nu_{ji}\,A_{ji}\int_{\mathrm{Disk}} N_j\,P^{esc}_{ji}\,dV\,, \tag{3}$$

where N_j is the number density of level n = j and A_{ji} is the spontaneous radiative transition probability (see Table 1). The results of this calculation are displayed in column 5 of Table 2. Note the large increase over equation (2), even with the P^{esc}_{ji} factor, to values which roughly agree with the observations of Ashok et al.(1984).

The Hα fluxes from 1 Del deserve comment. Both the diffuse and non-diffuse models match, by construction, the relative Hα line profile for 1 Del with the disk assumed to be seen edge on (Millar & Marlborough 1999a&b). However, the models predict different fluxes for other viewing angles and our current estimates for the *total flux* emitted from the entire envelope in Hα differ by a factor of $\sim$2.

3 DISCUSSION

As previously noted, typical observed values of Hα luminosities range from approximately 10^{34}erg s^{-1} for early type Be stars to 10^{32}erg s^{-1} for late type Be stars (Ashok et al.1984, Kastner & Mazzali 1989). Comparing to the fluxes listed in Table 2 which represent the total energy losses in these lines from the circumstellar envelope, we see that our best estimates for $F\,(\mathrm{H}\alpha)$, those computed with equation (3), are in good agreement with these observations for both early and late-type Be Stars. For the late-type Be stars, we are in disagreement with the theoretical flux estimates of AT who give $\sim$ 2x10^{29} erg s^{-1} as their best estimate for $\sim$ B8e. The

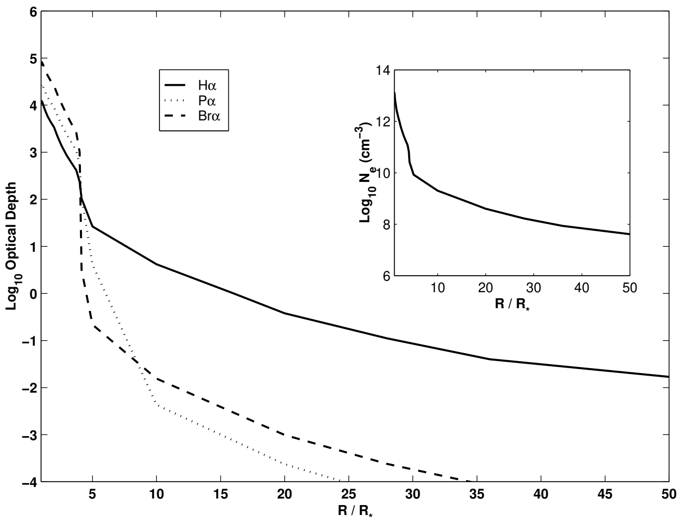

Figure 2. The log of the line centre optical depth, τ_o, for the indicated hydrogen transitions from the equatorial plane to the upper edge of the circumstellar disk for γ Cas as a function of distance from the rotation axis (R). Shown in the insert is the log of electron density in the equatorial plane.

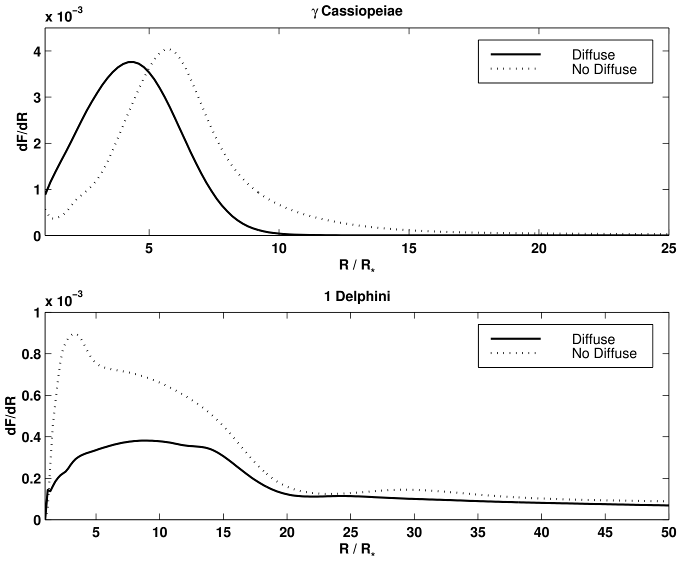

Figure 3. The radial derivative of the escape probability flux (equation 3) for γ Cas (upper panel) and 1 Del (lower panel) for models including (solid lines) and excluding (dotted lines) the diffuse component of the photoionizing radiation field. The area under each curve is the total flux escaping the envelope in Hα divided by ${R_*}^3$. The units of dF/dR are ergs cm^{-3} s^{-1}.

discrepancy is even larger if one compares with our flux estimate based on equation (1), a result which by construction, should be most comparable to AT. This indicates that we find an emission measure for the late Be disk that is much larger than AT. However, this flux estimate is a large overestimate of typically observed fluxes, and the inclusion of collisional excitation and optical depth effects are critically important to the formation of the Hα line.

One caveat in comparing our computed Hα fluxes with observations is that the observed absolute fluxes are obtained from measured Hα equivalent widths, $EW_{\mathrm{H}\alpha}$, for a particular (sometimes unknown) line of sight and are converted to absolute fluxes via

$$F\,(\mathrm{H}\alpha) \approx 4\pi R_*^2\, B_{\mathrm{H}\alpha}\,(T_*)\, EW_{\mathrm{H}\alpha}. \tag{4}$$

This conversion depends upon accurate radii, R_*, and continuum fluxes for the underlying stellar photospheres. In equation (4), the continuum photospheric flux, $B_{\mathrm{H}\alpha}$, is approximated by the Planck function at the wavelength of Hα for $T_* \approx T_{eff}$, as done by Ashok et al.(1984). In addition, equation (4) assumes that the stellar photosphere alone contributes to the continuum flux at Hα, an assumption which has been criticized by Stee et al.(1995) whose models indicate 10–20% contributions of the disk envelope to the continuum emission of Hα. Thus, there are large uncertainties involved in the transition from routinely observed equivalent width measurements to absolute line fluxes. Stee et al.(1998) and Kastner & Mazzali (1989) give values of the Hα luminosity for γ Cas of 6.36 x 10^{34} and 3.24 x 10^{34} erg s^{-1}, respectively with the difference in the values discussed by Stee et al.

The characteristic size of the Hα emitting region can now be constrained directly by interferometric observations which resolve the circumstellar disks of nearby Be Stars (Quirrenbach et al.1994, 1997; Stee et al.1995). In Figure 3, we plot the radial integrand, dF/dR, corresponding to equation (3) for the Hα fluxes of γ Cas and 1 Del. In the case of γ Cas, we see a maximal contribution at about 5 stellar radii. Quirrenbach et al. (1997) obtain $\sim$ 6 stellar radii as the FWHM of the Hα emitting region as based on Gaussian fits to observed visibilities as obtained by interferometric observations. The predicted contributions from 1 Del are much broader, giving significant contributions out to 10–15 stellar radii. There are no observations to compare with our models for 1 Del, although this radial extent does seem larger than expected on the basis of 1 Del's spectral type (see Quirrenbach et al.1997, Figure 8).

Comparing our results in Table 2 with these observations, we find no clear case for an additional source of ionizing radiation in order to produce the observed Hα emission for our particular choice of model parameters for either γ Cas or 1 Del. The discrepancies between Apparao's work and our calculations are due mainly to our realistic 2D geometry, and the inclusion of collisional excitation and optical depths in Hα. That case B recombination is inappropriate for the calculation of hydrogen line fluxes from Be envelopes has also been discussed by Hony et al.(1999). The importance of optical depths for line emission in Be stars is also discussed by van Kerkwijk et al.(1995).

Commensurate with these observational uncertainties are theoretical uncertainties in the calculation of the line fluxes. The escape probability approximation has been used, with each volume element assigned an escape probability based solely on the optical depth to the upper edge of the disk, along a ray perpendicular to the equatorial plane; this approximation is consistent with the treatment of radiative line losses in the underlying models. The enhancement of the escape probabilities due to the Doppler shifting of material by the macroscopic velocity field of the disk has been ignored, although inclusion of this effect on the line fluxes and the disk model itself would certainly be worthwhile. Nevertheless, we do not expect that these uncertainties will affect the basic conclusions of this work that the UV energy input of the underlying star is more than sufficient to explain the observed Hα fluxes, as opposed to failing to do so by several orders of magnitude as suggested by AT.

4 CONCLUSIONS

We have computed hydrogen line fluxes emitted in the important Hα, Pα, and Brα transitions by the circumstellar envelopes of early-type and late-type Be stars. These lines fluxes are based on models with self-consistent envelope temperatures which balance local energy loss and gain and which, individually, match the relative Hα line profiles of γ Cas and 1 Del. Using these models as typical of Be stars of their respective spectral types, we find that we can also successfully match the absolute Hα line fluxes of Ashok et al.(1984) for both early and late–type Be Stars. Our results are in disagreement with the suggestion of AT that an additional source of UV photoionizing radiation is required to explain the absolute Hα line fluxes from late-type Be stars. We find to the contrary that there is no clear case for an additional source of ionizing radiation. The observed Hα fluxes are consistent with the underlying stellar photosphere as the only source of photoionization energy input into the circumstellar disks.

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