Rates of Energy Gain and Loss in the Circumstellar Envelopes of Be Stars: the Disk Model

C. E. Millar[1,2] and J. M. Marlborough[1,3]

ABSTRACT

We have investigated the kinetic temperatures assumed by Waters et al. (1987) for the circumstellar envelopes of a group of Be stars. Waters (1986) introduced a simple equatorial disk model with $\rho(r) \propto r^{-n}$ to describe the physical characteristics of Be star disks. Since this equatorial disk model was applied to a large number of stars, it is important to ensure that the kinetic temperatures assumed by Waters et al. were reasonable. We have investigated their choice of kinetic temperature in two ways. Initially we tested the appropriateness of the temperatures assumed by evaluating the rates of energy gain and loss. Then, based on equating the rates of energy gain and loss, we determined self-consistently the temperature structure for these envelopes. Our investigation included the stars; γ Cas, δ Cen, ψ Per, and β CMi. For each of the stars examined we discovered that the constant temperature of the envelope assumed by Waters et al. was too high by a factor of 2 to 3.

Subject headings: circumstellar matter—stars: early-type—stars: emission-line, Be

[1]University of Western Ontario, Department of Physics and Astronomy, London, Ontario, Canada N6A 3K7

[2]cmillar@io.astro.uwo.ca

[3]marlboro@titan.astro.uwo.ca

1. Introduction

Be stars are rapidly rotating stars on or near the main sequence which have shown emission in the Balmer lines. It is thought that they are characterized by a two component wind. The UV absorption lines are asymmetric with extended short wavelength wings, suggesting a low density, high velocity (500 to 3000 km s^{-1}), radiatively driven wind that is approximately spherical (Zaal et al. 1997). Emission lines in the optical and infrared are formed in a disk-like, higher density wind with an outflow velocity of less than 100 km s^{-1} (Waters & Marlborough 1992). It is only this portion of the wind that is considered in this paper.

A simple cone-shaped, pure hydrogen disk model was introduced by Waters (1986) to describe the disk-like, equatorially concentrated wind. The density distribution is assumed to be given by the expression

$$\rho\,(\mathrm{r}) = \rho_{\circ}\{\mathrm{r}/\mathrm{R}_*\}^{-\mathrm{n}}, \qquad (1)$$

where ρ is the density, r is the radial distance, R_* is the stellar radius, $\rho_\circ$ is the density at $r = R_*$, and n is a number typically between 2 and 3.5. The density remains constant from the equatorial plane to the upper edge of the envelope on arcs of constant radial distance from the star. The velocity distribution is given by

$$\mathrm{v}\,(\mathrm{r}) = \mathrm{v}_{\circ}\{\mathrm{r}/\mathrm{R}_*\}^{\mathrm{n}-2}, \qquad (2)$$

where v is velocity at r and $v_\circ$ is velocity at R_*. The value of $v_\circ$ was assumed to be 5 kms^{-1} (Waters 1986).

This model has the advantage that it is described by relatively few free parameters. In addition to $\rho_\circ$, $v_\circ$, and n, the other parameters that must be specified are the inclination angle i of the rotation axis with respect to the observer's direction, the outer radius of the disk, R_{disk}, and the opening angle, Θ, of the disk. It is assumed that no material lies above the *upper edge* of the envelope which is defined as the boundary of the envelope that is at a maximum perpendicular distance from the equatorial plane as determined by the opening angle Θ.

In addition, and most important to this work, the kinetic temperature of the wind is a free parameter. The temperature determines the free-free and bound-free opacities and emissivities needed to interpret the IR excesses used in the curve of growth method to determine the wind parameters (Waters et al. 1987). These models have been used to derive the density structure and mass loss rates for Be star disks using data from IRAS (Waters et al. 1987, Waters 1986, Dougherty et al. 1994). Waters et al. *assumed* $T_{wind} = 0.8T_{eff}$. It is important to check the envelope temperatures assumed in order to assess the appropriateness of the disk model. In addition, matching the hydrogen profiles, such as H_α, provides important constraints on the wind parameters. A self-consistent temperature at each position in the envelope is necessary to predict accurate line profiles (Millar & Marlborough 1999a&b).

In order to test the relevance of the wind temperatures assumed by Waters et al. (1987) we have calculated, using the identical wind parameters of Waters, the rate energy gain and energy loss throughout the envelope and then, by balancing these rates, determined a self-consistent temperature at various positions in the envelope. We purposely chose the same parameters as Waters so that we could test the suitability of the constant temperatures assumed. Once the values of $\rho_\circ$, θ, and v_o are assigned for a specific model, the mass loss rate is determined via the equation of continuity. Thus, the mass loss rates for our variable temperature models are the same as those determined by Waters (1987).

We have examined the disk model for 4 Be stars which have a range in spectral type: γ Cas (B0IVe), δ Cen (B2IVne), ψ Per (B5Ve), and β CMi (B8Ve). We have tested the validity of the envelope temperature assumed by Waters (1986) for each star and, more importantly, we have determined the variation of temperature throughout the envelope from the requirement of equal rates of radiative energy gain and radiative energy loss at each location.

2. Calculations

The rate of radiative energy gain and loss in the pure hydrogen gas was determined at various locations in the circumstellar envelope. At any position a unit volume of gas gains energy from photoionization, collisional de-excitation of excited levels and free-free absorption; it loses energy from recombination, collisional excitation and free-free emission. In a steady state, the kinetic temperature of the gas at a particular location is that temperature for which the rates of energy gain and loss are equal. Additional details may be found in Millar & Marlborough (1998).

We have calculated the ratio of energy gain to loss at various locations in the envelope using the same stellar parameters, including the isothermal envelope temperature assumed by Waters et al. (1987). If, by chance, the kinetic temperature assumed by Waters et al. were correct, the energy ratios would equal unity everywhere in the envelope. A simple global average of this set of ratios provides a measure of how good or otherwise was the temperature assumed initially. A density weighted average of the energy ratio provides information about the assumed temperature in areas of the envelope where densities are high.

Subsequently, the program was modified to allow a temperature to be specified at each location considered in the envelope. For each star, the kinetic temperature assumed by Waters et al. (1987) was used as the initial guess at each location. The ionization-excitation program predicted the ratio of energy gain to loss for each grid location. Then the temperature at a particular location was adjusted, in steps of ± 500 K depending of the value of the ratio there. The program was then rerun using the modified temperature grid and the energy gain to loss ratios recalculated at each location. This iterative process was repeated until the ratio of gain to loss at each location considered was $1 \pm .02$. However, the individual temperatures are certainly not this accurate. The error in the level populations is approximately 5% due to the method by which they were calculated; this will result in errors in the temperature of approximately 10%.

3. Results

3.1. Isothermal Envelope Results

The ratio of energy gain to energy loss for the isothermal envelope models assumed by Waters et al. (1987) are given in Table 1. For all of the stars investigated, the average of the ratio of energy gain to energy loss is less than one implying that the temperatures assumed by Waters were all too large. In order to achieve equal rates, the assumed wind temperatures must be reduced.

The results for both the simple global average ratio and the density weighted average ratio are very similar for each star. This simply reflects the fact that at a particular radial position the disk model has a density distribution which varies very slowly with increasing distance perpendicular to the equatorial plane. This is in contrast to the Poeckert Marlborough model (1978), hereafter PM, in which the density distribution decreases exponentially perpendicular to the equatorial plane.

3.2. Results for a Variable Temperature Envelope

The temperature was calculated at 480 predetermined locations within the envelope. Twenty four radial positions were chosen so that there are more grid points close to the star where the density is the highest. At each radial position, 20 grid points (called J values) perpendicular to the equatorial plane were co It is important to realize that the J values which correspond to the mid-position of the disk are not at a constant height above the equatorial plane, but instead are at an increasing distance from the equatorial plane, since the disk flares with increasing radial distance. A schematic diagram of the disk model is shown in Waters (1987). The final set of temperatures for each star corresponding to equal rates of energy gain and loss as a function of radial distance from the star for 3 locations in the envelope, the equatorial plane, (J= 1), the upper edge, (J= 19), and mid-way between the equatorial plane and upper edge are displayed graphically in Figures 1, 2, 3, and 4, respectively. It is apparent from these figures that there are portions of the envelope where the temperature is approximately constant. However, there are differences in detail among the 4 stars investigated which are attributed to differences in the assumed $\rho_\circ$ and the effective temperature of the star.

Basically as a first order approximation, the temperature in the equatorial plane is fairly constant for all stars investigated. With increasing height above the equatorial plane, the temperature increases at locations where optical depths are not too large, generally at radial distances closer to the star; at larger radial distances the temperature may decrease due to both increasing optical depths and the geometrical dilution of stellar radiation. This effect is most pronounced for the coolest stars which have fewer ionizing continuum photons. Eventually, since the lines of sight gradually traverse less material near the uppermost edge, the temperature steadily increases toward the upper edge of the envelope at all radial distances.

Generally, the upper edge is the hottest portion of the envelope. The exception is β CMi which, after some temperature variations near the star, has a constant temperature of approximately 5,000 K throughout the envelope out to a distance of 50 stellar radii; the temperature then drops by approximately 2,000 K along the upper edge. β CMi has an effective tem-

Table 1: Energy Ratios for Isothermal Models

HD	HR	Name	Spectral Type	T Wind [K]	Global Average of Ratios ($\pm 1\sigma$)	Density Weighted Average of Ratios ($\pm 1\sigma$)
5394	193	γ Cas	B0IVe	25000	0.56±.38	0.598±.003
105435	4621	δ Cen	B2IVne	16000	0.30±.13	0.368±.002
22192	1087	ψ Per	B5Ve	12000	0.26±.09	0.304±.001
58715	2845	β CMi	B8Ve	10000	0.17±.05	0.174±.001

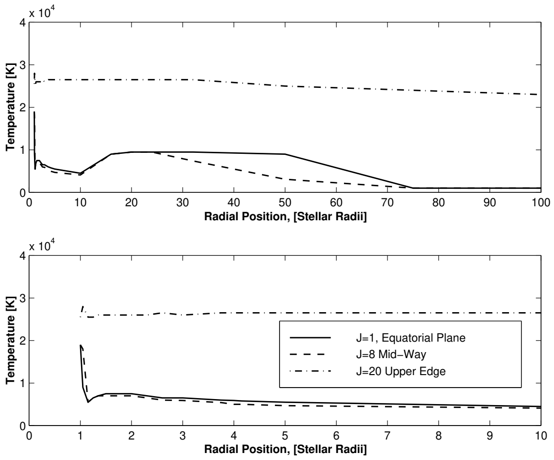

Fig. 1.— The temperature as a function of distance from the rotation axis for which the rate of energy gain balances the rate of energy loss for the star Gamma Cas for three locations in the circumstellar envelope: the equatorial plane (J=1), mid-way, and the upper edge.

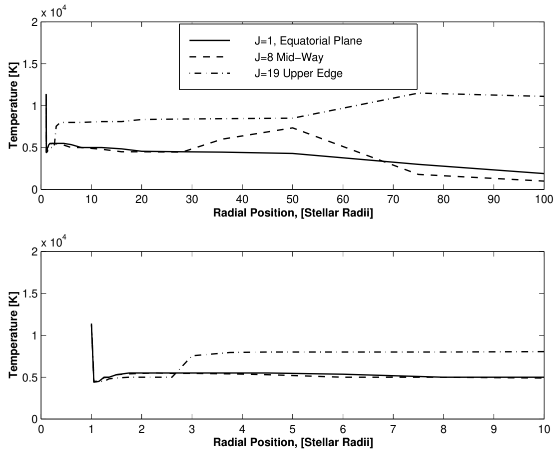

Fig. 2.— The same as Figure 1 for the star Del Cen.

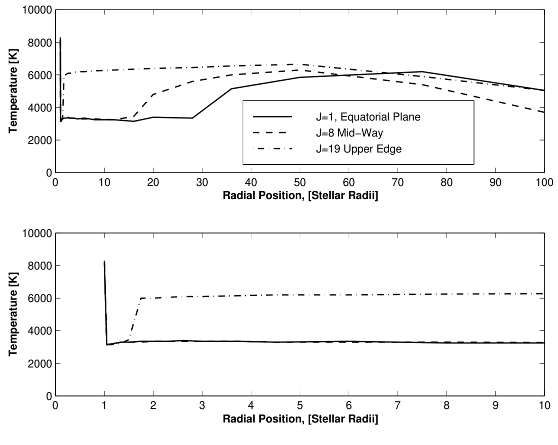

Fig. 3.— The same as Figure 1 for the star Psi Per.

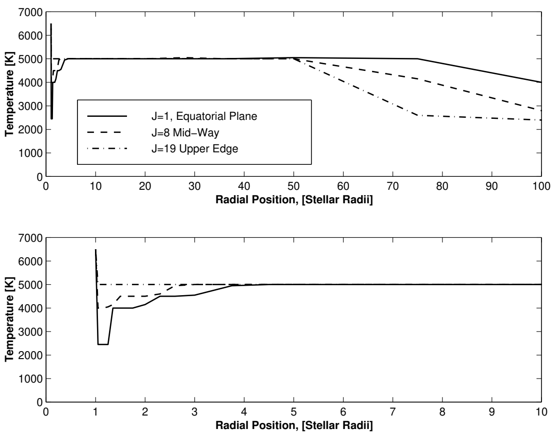

Fig. 4.— The same as Figure 1 for the star Beta CMi.

perature of 12000 K (Waters et al. 1987). The other stars, γ Cas, δ Cen, and ψ Per have effective temperatures of 31000 K, 20000 K, and 15000K, respectively. If the effective temperature of β CMi were increased by a few thousand degrees K, then the upper edge of the envelope would become hotter than the interior under the condition of equal rates of energy gain and loss. For β CMi, the central star simply does not have enough ionizing photons to keep the outer edge hotter than the interior. Similarly for the B5 star, ψ Per, there is a slight decrease in temperature at large radial distances along the upper edge; see Figure 3. However, this decrease is not as pronounced for ψ Per as for β CMi, the coolest of the objects investigated.

The disc model has constant densities on arcs of constant radial distances from the star. Therefore, the density inside the envelope does not change significantly with increasing height above the equatorial plane. Consequently, the processes that are important in one part of the envelope are typically important elsewhere. As a result, although there are some temperature variations within the envelopes of the stars investigated and some differences between the stars investigated due to different effective temperatures and densities of the envelope, the temperatures within the envelope vary gradually. This is noticeably different from the PM model where the exponential decrease in density above the equatorial plane resulted in large temperature variations within the envelope (Millar & Marlborough 1998). As a comparison, the density distribution along the equatorial plane in the disk model for γ Cas is approximately an order of magnitude *lower* than that assumed for the PM model. However, since the density falls off exponentially with increasing height above the equatorial plane in the PM model, the density for the disk model at the upper edge of the envelope, is approximately an order of magnitude *higher* than PM's density.

The outer edge of the envelope for Gamma Cas is quite hot due to the high effective temperature of the star. Adjacent to the uppermost edge, just inside the envelope there are differences in optical depths which result in temperature fluctuations due to rapidly changing conditions. These variations are unphysical and are due to large differences in optical depths along that part of the lines of sight considered between the assumed boundary of the upper edge, J = 20, and the next adjacent location considered, J = 19, in the z direction. Figure 1 shows the variation of the temperature with distance from the rotation axis for the J = 20 position. The model assumes that the envelope is optically thin in all lines along the upper edge, that is, at J = 20 and that there is no material beyond this assumed boundary. Therefore, this boundary is somewhat unphysical. Along the outer edge of the envelope, photons can escape freely into space. However, since the density remains constant along lines of constant radial distance from the star and since Gamma Cas has the largest assumed $\rho_\circ$ of the stars investigated, at the J = 19 position, the envelope is optically thick in all lines out to a radial distance of 2.75 stellar radii and optically thick in Lyman lines out to a radial distance of 20 stellar radii. Hence, between J = 20 and J = 19, conditions in the envelope change significantly. At a distance of 20 stellar radii the envelope becomes optically thin in all lines and a relatively constant temperature is maintained with increasing radial distance throughout the upper sections of the envelope.

3.3. Sources of Energy Gain and Loss

Energy gain and loss are dominated by photoionzation and radiative recombination at all locations of the envelope for all 4 stars investigated. Photoionization is the dominant source of energy gain to the gas for each star. Densities are never high enough for collisional energy terms to dominate. Nevertheless, collisional processes are important in populating upper energy levels from which electrons may then be ejected by photoionization. A radiative excitation of a bound electron followed by a collisional de-excitation represents a net energy gain to the gas. Conversely, a collisional excitation followed by a radiative de-excitation will result in an energy loss. Since the density is constant along radial arcs there is a modest reduction in density with increasing height above the equatorial plane. As a result, collisional processes are somewhat more important in the equatorial plane than at the upper edge. Generally, for all 4 stars collisional energy gain (collisional de-excitation) is greater than collisional energy loss. For δ Cen and γ Cas, however, energy loss is greater than energy gain due to collisions at the upper edge of the envelope. These 2 stars have the higher effective temperatures. The collisional transition rate coefficient for the transition from level 1 to 2 for the model hydrogen atom considered and the collisional transitions allowed in this model, is the most strongly affected by temperature. As a result, the higher kinetic temperatures due to the higher effective temperatures of these stars greatly increase this coefficient and consequently the total amount of collisional energy loss is increased.

For γ Cas, the main source of photoionization energy in the equatorial plane comes from levels 2p and 2s out to a distance of approximately 65 stellar radii. At this distance from the star the photoionization energy from level 2p drops significantly. From this distance to the upper edge of the envelope, photoionization energy is dominated by levels 2s and n = 1. The number density of level 2p decreases due to the gradual decrease in total number density and the corresponding reduction in importance of collisional processes with the result that fewer electrons can get into the higher levels. Also, since level 2s is metastable, its number density builds up so that this level provides a larger proportion of the photoionization energy. Along the upper edge of the envelope, ionization from levels 1 and 2s contributes the most energy from the gas.

The dominate source of energy gain along the equatorial plane for the star δ Cen is photoionization from level 2p. Photoionization from levels 2s and $n = 3$ is also significant. Along the upper edge of the envelope, energy gain is dominated by photonization from 2p and 2s to a distance of approximately 60 stellar radii. At greater radial distances, energy gain is dominated by photoionization from level n = 1 along the upper edge.

The relative contributions to the energy gain by photoionization for ψ Per are the same as δ Cen both at the upper edge and in the equatorial plane. Along the upper edge, however, energy gain is dominated by photoionization from level n = 1 at radial distances greater than 65 stellar radii. The photoionization energy for β CMi comes from similar atomic levels as both δ Cen and ψ Per. However, along the equatorial plane, photoionization from level 3 only contributes out to a distance of approximately 3 stellar radii and then drops off rapidly in importance. This is due to a lower level population in level n = 3 which is a direct consequence of the lower density of the envelope of β CMi.

4. Conclusions

It is clear from Table 1 that the temperatures assumed by Waters for the isothermal disks of these Be stars were too high. In order to obtain equal energy gain and loss rates it is necessary to lower the wind temperature. In a recent parameter study of the disk model for the Be star ψ Per, the line shapes and ratios of line strengths were found to be strongly dependent on the temperature structure of the envelope (Marlborough, Zijlstra, & Waters 1997). Therefore, a self-consistent temperature distribution is an essential step to determine the physical parameters that describe conditions in the envelopes of Be stars. Figures 1, 2, 3, and 4 show the self-consistent temperature structure for the stars investigated. The differences in the temperature of the 4 stars with equal energy rates are due to differences in the assumed $\rho_\circ$ and in the effective temperatures of the stars. It is possible to determine a self-consistent set of temperatures which correspond to a given value of $\rho_\circ$ in order to match emission line shapes and ratios of the line strengths.

The envelope temperature distribution sets the continuum flux in the IR which may alter the value of the derived $\rho_\circ$ via the curve of growth method of Waters et al. (1987). The derived $\rho_\circ$ directly affects the hydrogen level populations as well as the model predictions of mass loss rates. Recall, the mass loss rates are set by the density and velocity distributions. However, the greatest uncertainty in the mass loss rates is due to the uncertainty in the assumed initial velocity, v_o. Unfortunately, the velocity distributions within Be star disks are not well known and an increase in v_o will increase the mass loss rate.

The disk model has been used successfully to model the IR excess of Be stars. Dougherty et al. (1994) demonstrate the importance of the temperature distribution for emission line shapes and relative line strengths. Also, the temperature distribution in the envelope sets the ionization fraction. Knowledge of the ionization gradients within the envelope may prove to be a valuable probe of disk structure. Therefore, the approach used here, which determines the temperature self-consistently, is essential to decipher the characteristics of these disk-like envelopes. Self-consistent temperatures eliminate one additional degree of freedom in Be star models. The remaining parameters may then be determined with greater confidence.

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