The Formation of Protostellar Disks in Magnetized Cloud Cores

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Abstract. We use magnetic collapse models to place some constraints on the properties of a centrifugally-supported inner disk that is formed from the collapse of a magnetic core. The core represents an outer nonequilibrium flattened envelope which collapses dynamically to form a protostar and inner disk. The inner disk forms after a central protostar has formed, and grows by dynamical accretion from the envelope. We find that the centrifugal radius of the inner disk is related to its mass by the two important parameters characterizing the background medium: the background rotation rate $\Omega_{\rm b}$ and the background magnetic field strength $B_{\rm ref}$. To form a star-disk system of the type that is typically observed requires significant redistribution of mass and angular momentum within the inner disk. As the central star gains mass at the expense of the disk, the disk radius also increases dramatically if the total angular momentum is conserved.

INTRODUCTION

Disks around young stellar objects are thought be a natural consequence of the collapse of a rotating interstellar cloud. In this paper, we use a semianalytic collapse model for a rotating, magnetized cloud core [1] to find a connection between ambient cloud properties and the size of an inner centrifugally-supported disk that is formed by the collapse.

FORMATION OF A CENTRIFUGAL DISK

Star formation will occur within a flattened envelope in the presence of dynamically significant magnetic fields [2]. While maintaining near-equilibrium along the magnetic field lines, an initially subcritical (i.e., mass-to-flux ratio below a critical value) cloud will contract radially toward local density peaks due to ambipolar diffusion, but the contraction will become dynamic within a central region when it has achieved a supercritical mass-to-flux ratio [2–4]. The collapse of the inner region of the flattened core leads to power-law radial profiles in the surface density, angular velocity, and magnetic field strength. A natural limiting form for isothermal collapse is when the power-law profile extends inward to radius $r = 0$, creating a central singularity. Since a finite mass now exists at $r = 0$, this moment (labeled $t = 0$) is usually associated physically with the formation of a central protostar. The magnetic collapse models typically lead to the limiting column density profile

$$\sigma(r) \simeq \frac{c_{\rm s}^2}{Gr} \tag{1}$$

[1], where $c_{\rm s}$ is the isothermal sound speed and G is the universal gravitational constant. The angular velocity Ω achieves the limiting profile

$$\Omega(r) \simeq \frac{2\pi\Omega_{\rm b} c_{\rm s}^2}{B_{\rm ref} G^{1/2} r} \tag{2}$$

[1], where $\Omega_{\rm b}$ is the ambient rotation rate of the cloud and $B_{\rm ref}$ is a uniform background, or "reference", magnetic field. These values are important because magnetic braking enforces $\Omega \simeq \Omega_{\rm b}$ until the column density achieves the critical value $\sigma_{\rm crit} \simeq 2B_{\rm ref}/(2\pi G^{1/2})$ for collapse [4,1]. The subsequent dynamic collapse is characterized by near angular momentum conservation, hence the relation $\Omega \propto \sigma$. Equations (1) and (2) yield the following relation between specific angular momentum $j = \Omega\, r^2$ and the enclosed mass m,

$$j \simeq \frac{\Omega_{\rm b} G^{1/2}}{B_{\rm ref}}\, m. \tag{3}$$

This $j - m$ relation is preserved during the dynamic collapse phase before a central protostar is formed ($t < 0$) and also after protostar formation ($t > 0$), when the collapse becomes even more dynamic in an inner region where the infall resembles free-fall onto a central point mass. During the time $t < 0$, infalling mass shells near the cloud center cannot hit a centrifugal barrier even if angular momentum is conserved, due to the nature of self-gravity in a self-similarly collapsing thin disk [5,1]. However, during the phase of dynamical accretion onto a central point mass ($t > 0$), we can use the relation $g_r \simeq -Gm/r^2$ for the gravitational acceleration (the equality is exact for a spherical cloud). Hence, each mass shell (of fixed j and m) will hit a centrifugal barrier at the radius $r_{\rm c}$ where

$$\frac{j^2}{r_{\rm c}^3} \simeq \frac{Gm}{r_{\rm c}^2}. \tag{4}$$

Incorporating equation (3), we see that the centrifugal radius is

$$r_{\rm c} \simeq \left(\frac{\Omega_{\rm b}}{B_{\rm ref}}\right)^2 m. \tag{5}$$

Using some standard values for molecular clouds, we find that

$$r_{\rm c} \simeq 15 \left(\frac{\Omega_{\rm b}}{10^{-14}\,{\rm rad\,s^{-1}}}\right)^2 \left(\frac{30\,\mu{\rm G}}{B_{\rm ref}}\right)^2 \left(\frac{m}{1\,M_\odot}\right) {\rm AU}. \tag{6}$$

The squared dependence on $\Omega_{\rm b}$ and $B_{\rm ref}$ allows for a considerable range in values of $r_{\rm c}$.

The mass m of the star-disk system will grow by dynamical accretion from the infalling envelope. The mass accretion rate will be of the form $\dot{m} = m_0\, c_{\rm s}^3/G$, where m_0 is a constant if the solution is strictly self-similar. In reality, boundary effects act to limit the infall velocity of outer mass shells, and cause $m_0 = m_0(t)$ to be a monotonically decreasing function of time through the accretion phase [1]. The semianalytic collapse model [1] shows that $m_0(0)$ will most likely fall in the range 10-20. The accretion will ultimately have to be shut off due to either a limited mass reservoir or to some back reaction from the star, e.g., an outflow.

EVOLUTION OF THE DISK

A centrifugal disk that is formed in the manner described above, with radius $r_{\rm c}$ given by equation (5), will contain most of its matter in the outer disk. Only a very small percentage of the mass will fall directly onto the star. Hence, the creation of typical stars requires that most mass shells shed the bulk of their angular momentum and accrete onto the central star. Furthermore, estimates of disk masses around young stellar objects are usually about an order of magnitude below the estimated stellar mass [6,7], implying that the mass transfer is relatively efficient. The required redistribution of mass may occur if the disk loses angular momentum to its surroundings, e.g., through a disk wind, or it may occur due to internal redistribution of mass and angular momentum by gravitational and/or viscous torques. In the latter case, transport of the bulk of the mass to the central star also requires that some mass move outward to carry the angular momentum of the system. The end result is that the size of the disk increases dramatically, even though it no longer contains most of the system mass. This evolutionary scenario is consistent with the general tendency of any disk of a given angular momentum to achieve its lowest energy state when all of the mass is brought to $r = 0$ and an infinitesimal mass carries all of the angular momentum at an infinite radius [8].

In the remainder of this paper, we derive an estimate of the final disk radius if accretion occurs due to internal torques. We write $m = m_{\rm s} + m_{\rm d}$, where $m_{\rm s}$ is the mass of the star and $m_{\rm d}$ is the mass of the disk. If $m_{\rm s} \gg m_{\rm d}$, as suggested by observations, then the Keplerian angular velocity in the disk is

$\Omega = (Gm_{\rm s})^{1/2}/r^{3/2}$. Consequently, if the disk of mass $m_{\rm d}$ contains essentially all the angular momentum J, then its radius is

$$r_{\rm cf} = \frac{\ell}{Gm_{\rm s}} \left(\frac{J}{m_{\rm d}} \right)^2 , \tag{7}$$

where ℓ is a constant of order unity which depends on the assumed column density profile in the disk. For the time being, we assume the dependence $\sigma \propto r^{-3/2}$, which is the inferred profile for the protosolar nebula [9]. In this case $\ell = 4$, and if we integrate equation (3) to find the total angular momentum J, the previous relation can be rewritten as

$$r_{\rm cf} \simeq \left(\frac{\Omega_{\rm b}}{B_{\rm ref}} \right)^2 \left(\frac{m}{m_{\rm s}} \right) \left(\frac{m}{m_{\rm d}} \right)^2 m. \tag{8}$$

For the case $m \approx m_{\rm s} \gg m_{\rm d}$, and using equation (5), we find

$$r_{\rm cf} \simeq r_{\rm c} \left(\frac{m}{m_{\rm d}} \right)^2 . \tag{9}$$

Normalizing to typical values yields

$$r_{\rm cf} \simeq 1500 \left(\frac{\Omega_{\rm b}}{10^{-14}\,{\rm rad\,s^{-1}}} \right)^2 \left(\frac{30\,\mu{\rm G}}{B_{\rm ref}} \right)^2 \left(\frac{m}{1\,M_{\odot}} \right) \left(\frac{m/m_{\rm d}}{10} \right)^2 {\rm AU}. \tag{10}$$

Equations (5) and (9) bracket the possible radii for protostellar disks in star-disk systems of mass m. Dynamical accretion from a larger infalling envelope tends to build up a centrifugal disk of size $r_{\rm c}$, but the disk size tends toward the value $r_{\rm cf}$ if the disk accretion is driven largely by internal redistribution of angular momentum.

REFERENCES

1. Basu, S. 1997, ApJ, 485, 240
2. Fiedler, R. A., & Mouschovias, T. Ch. 1993, ApJ, 415, 680
3. Ciolek, G. E., & Mouschovias, T. Ch. 1994, ApJ, 425, 142
4. Basu, S., & Mouschovias, T. Ch. 1994, ApJ, 432, 270
5. Basu, S., & Mouschovias, T. Ch. 1995, ApJ, 452, 386
6. Beckwith, S. V. W., & Sargent, A. I. 1993, in Protostars and Planets III, eds. E. H. Levy & J. I. Lunine, (Univ. of Arizona Press: Tucson), 521
7. Mundy, L. G. 1997, in Star Formation Near and Far, eds. S. S. Holt & L. G. Mundy, (AIP: Woodbury), 63
8. Lynden-Bell, D., & Pringle, J. E. 1974, MNRAS, 168, 603
9. Weidenschilling, S. J. 1977, Ap&SS, 51, 153