# Self-Similar Evolution of Supercritical Cores

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**Abstract.** We use a semi-analytic model to examine the collapse of supercritical cores (i.e., cores with a mass-to-flux ratio exceeding a critical value). Recent numerical simulations of the formation and contraction of supercritical cores show that the inner solution tends toward self-similar evolution. We use this feature to develop analytic expressions for quantities such as the density, angular velocity, and magnetic field. All forces involved in the problem (e.g., gravitational, magnetic, thermal, and centrifugal) can be calculated analytically in the thin-disk geometry of the problem. The role of each force during the contraction is analyzed, and we identify the key role of ambipolar diffusion in accelerating the collapse. We find that the collapse is dynamic and supersonic velocities are achieved in the innermost region of the core by the time of protostar formation. The mass accretion rate is significantly greater than the canonical $C^3/G$ at the moment of protostar formation, although we argue that it is time-dependent and will eventually decrease. Comparisons are made with the predictions of existing spherical similarity solutions.

## 1. INTRODUCTION

The collapse of gravitationally unstable objects is a central problem in the theory of star formation. Similarity solutions play an important role in understanding gravitational collapse since they provide a simple analytical description of the evolution. Larson (1969) showed that the approach to protostar formation in his numerical simulations could be described by an isothermal similarity solution. This similarity solution, also developed by Penston (1969), is known as the Larson-Penston (LP) solution. A different similarity solution has been found by Shu (1977), which begins at the moment of protostar formation (identified with the presence of a central density singularity and labeled as $t = 0$ in both solutions) and describes the subsequent accretion onto the central point mass. The two similarity solutions differ greatly in their description of the protostellar environment at $t = 0$ (see § 4). However, both solutions assume spherical symmetry and do not include the effects of magnetic fields or rotation. In this paper, we estimate the conditions at $t = 0$ when these sources of support are taken into account. We use the results of detailed magnetohydrodynamic (MHD) numerical simulations (Fiedler & Mouschovias 1993; Ciolek & Mouschovias 1994, hereafter CM94; Basu & Mouschovias 1994, hereafter BM94) to develop analytic expressions for important physical quantities. We find that magnetic fields *cannot* enforce a near-quasistatic approach to protostar formation, due to the effect of ambipolar diffusion, and that the maximum velocity in the innermost region is supersonic. This means that the inner solution is qualitatively more similar to the LP solution for spherical collapse. We find that the mass accretion rate onto the protostar is much higher than in the Shu solution, although we expect that it is time-dependent and will eventually decrease.

## 2. SELF-SIMILAR PROFILES

Molecular clouds with initially subcritical mass-to-flux ratios (i.e., magnetically dominated) tend to flatten along the (vertical, or $z$) direction of the ambient magnetic field, and evolve due to ambipolar diffusion, the drift of neutral particles relative to charged species. When a supercritical region (core) is formed, gravitational contraction proceeds more rapidly, and the inner core

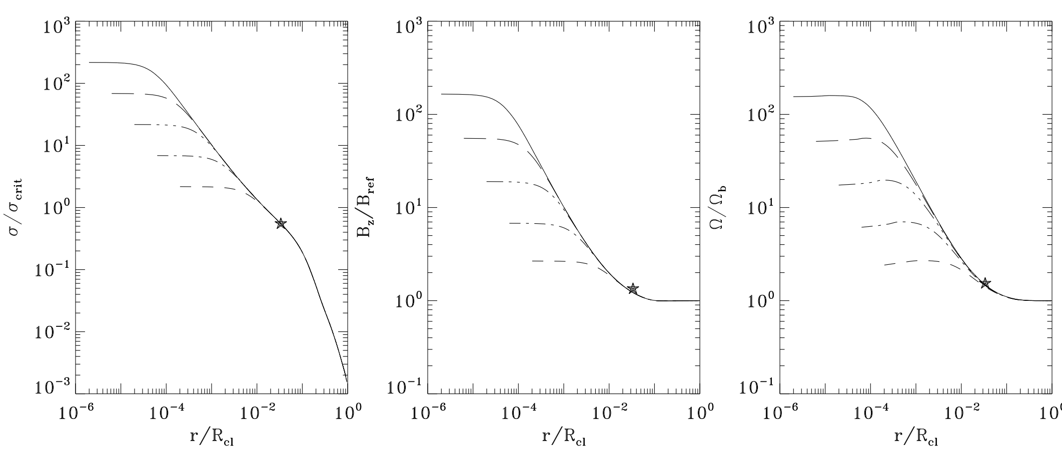


**FIGURE 1.** Spatial profiles of physical quantities at five different times following the formation of a supercritical core, in the standard model of BM94. The star marks the boundary of the supercritical core. The column density $\sigma$ is normalized to the value $\sigma_{\rm crit}$ at which a critical central mass-to-flux ratio is achieved, the vertical magnetic field $B_z$ is normalized to the ambient field $B_{\rm ref}$, and the angular velocity $\Omega$ is normalized to the background value $\Omega_{\rm b}$. All are plotted versus the radius $r$ normalized to the initial cloud radius $R_{\rm cl} = 5.76$ pc. The inner profiles tend to evolve in a self-similar manner.

ultimately evolves through a series of self-similar profiles. Figure 1 shows the evolution of the supercritical core in the standard model of BM94.

The self-similarity of the inner core takes the following form. The (vertically integrated) column density $\sigma$, vertical component of the magnetic field $B_z$, and angular velocity $\Omega$ are described by

$$\sigma(r,t) = \sigma_{\rm c}(t) \Big/ \sqrt{1+(r/R)^2}, \tag{1}$$

$$B_z(r,t) = 2\pi\sqrt{G}\,\mu^{-1}\,\sigma_{\rm c}(t) \Big/ \sqrt{1+(r/R)^2}, \tag{2}$$

$$\Omega(r,t) = 2\Omega_{\rm c}(t)\left(\frac{R}{r}\right)^2\left[\sqrt{1+(r/R)^2}-1\right], \tag{3}$$

where $R = R(t)$ is the scale factor. Equation (2) is obtained from equation (1) since $\mu$, the mass-to-flux ratio in units of the critical value $(2\pi\sqrt{G})^{-1}$ for a uniform disk (Nakano & Nakamura 1978), is spatially uniform in the inner region (CM94; BM94). Equation (3) follows from equation (1) since the specific angular momentum is a linear function of the enclosed mass (BM94). Angular momentum conservation implies $\Omega_{\rm c}(t) = \Omega_{\rm b}\,\sigma_{\rm c}(t)/\sigma_{\rm crit}$, where $\Omega_{\rm b}$ is the rotation rate of the background medium and $\sigma_{\rm crit}$ is the column density at which the central flux tube achieves a critical mass-to-flux ratio.

The scale factor $R(t)$ measures the size of a central near-uniform column density region. Physically, it is the region in which thermal-pressure plays a significant role and helps to smooth out any density inhomogeneities. Therefore, we can relate it to the critical thermal length scale $\lambda_{\rm T,cr}$ for a thin disk (see discussion in CM94 and BM94), i.e.,

$$R(t) = \ell\lambda_{\rm T,cr} \equiv \ell\, C^2/2G\sigma_{\rm c}(t), \tag{4}$$

where $C$ is the isothermal sound speed. The numerical simulations of BM94 show that $\ell \simeq 2$ in the late stages of collapse.

Equations (1)-(3) give excellent fits to the inner profiles of the supercritical cores calculated by BM94, and allow an analytic study of the central evolution.

## 3. APPROACH TO PROTOSTAR FORMATION

The analytic profiles given above allow a calculation of all forces in the inner region of the thin-disk cloud, using the formulas given by CM93 and BM94. We find that the ratio of thermal-pressure to gravitational acceleration $a_{\rm T}/|g_r|$ stays constant during the collapse, and is equal to $1/\pi = 0.32$ in the central region. The central centrifugal support $a_{\rm c}/|g_r|$ also stays constant to the extent that angular momentum is conserved during the supercritical phase. However, it equals $\sim 10^{-5}$, so that centrifugal support is negligible during this stage. The central magnetic support is $a_{\rm M}/|g_r| = (1+2/\pi)/\mu^2$ and is also constant during collapse if the central mass-to-flux ratio $\mu$ is fixed (flux-freezing). However, ambipolar diffusion continues to play a slow but significant role in the supercritical phase, causing an increase in $\mu$ of the form

$$\mu \propto \sigma_{\rm c}^{\epsilon}, \tag{5}$$

where $\epsilon \simeq 0.05$ in the standard model of BM94. This is significant enough to eventually reduce the once dominant magnetic force to a secondary source of support to thermal-pressure in the inner core (see quantitative discussion in Basu 1997). However, the magnetic forces continue to be the dominant source of support in the outer core and certainly in the subcritical envelope.

We can also estimate the infall velocity $v_r$ (consistent with self-similarity) in the manner originally used by Narita, Hayashi, & Miyama (1984) to analyze self-similar evolution in rotating, nonmagnetic clouds: 1) we use the mass continuity equation with the column density profile of equation (1) to find

$$v_r = \frac{R}{r}\left(\sqrt{1+r^2/R^2}-1\right)\dot{R}, \tag{6}$$

and 2) we differentiate the above expression and equate (to first order in $r/R$) to the analytically calculated accelerations obtained from equations (1)-(3). Taking the limit of protostar formation ($\sigma_{\rm c} \rightarrow \infty$ or $R \rightarrow 0$), when thermal-pressure forces provide the dominant central support, we find

$$\dot{R} = -\sqrt{2(\pi-1)}\; C = -2.07\; C. \tag{7}$$

Equation (6) shows that $v_r$ tends to a constant value $v_r = \dot{R}$ for $r \gg R$. In the limit that $R \rightarrow 0$, $v_r$ has this value at all radii. Thus, the mass accretion rate at protostar formation is

$$\dot{M} = -2\pi\sigma r v_r = 2\pi\frac{\sigma_{\rm c}R}{r}\; r\; 2.07\; C = 13\;\frac{C^3}{G}, \tag{8}$$

where we have used equation (4) to replace the constant value $\sigma_{\rm c}R$.

## 4. CONCLUSIONS

The study of similarity solutions for spherical, isothermal collapse have found two very different scenarios. The solution of Shu (1977) envisages a slow, quasistatic approach to protostar formation ($t = 0$), followed by a dynamic inside-out collapse of the cloud onto the protostar. There is negligible acceleration before the stellar core forms and the subsequent mass accretion rate is $\dot{M} = 0.975\, C^3/G$. In the LP solution, the contraction is dynamic as $t = 0$ is approached (net central acceleration $a_{\rm tot} = -0.4|g_r|$) and the infall velocity is $v_r = -3.3\, C$ at all radii at $t = 0$. The mass accretion rate is

$\dot{M} = 29\,C^3/G$ at $t = 0$ and rises to $\dot{M} = 47\,C^3/G$ for $t > 0$ (Hunter 1977), when a free-fall flow is established in the vicinity of the protostar.

Our semi-analytic model, which is based on numerical simulations which realistically model the effects of magnetic fields and rotation, allows us to estimate core properties during the approach to protostar formation. Our conclusions are: 1) The inner region achieves considerably dynamic contraction *before* a protostar is formed. The central acceleration tends to the limiting value $a_{\rm tot} = -0.68|g_r|$ as magnetic support is lost due to the relatively slow but effective magnetic flux diffusion in the inner region. 2) The infall velocity is supersonic, and tends to the limiting value $v_r = -2.07\,C$. 3) The mass accretion rate at $t = 0$ approaches

$$\dot{M} = 13\frac{C^3}{G} = 13(1.6 \times 10^{-6}) \left[\frac{C}{0.19\,{\rm km\ s^{-1}}}\right]^3 \frac{M_\odot}{\rm yr}. \tag{9}$$

Therefore, the collapse in the inner region of our model is qualitatively more similar to the LP solution, though there are quantitative differences.

We believe that our model agrees with observations which suggest that the mass accretion rate around a newly formed protostar (a Class 0 object) is considerably higher than $C^3/G$ (Ward-Thompson 1996; Bontemps et al. 1996). Ward-Thompson (1996) estimates a mass accretion rate of $\sim 10^{-5} M_\odot$ yr $^{-1}$ in the Class 0 stage, in agreement with our estimate in equation (9). The estimated accretion rate at the later Class I phase is $\sim 10^{-6} M_\odot$ yr $^{-1}$ and implies that $\dot{M}$ decreases with time following protostar formation. This is inconsistent with all of the similarity solutions, which require that the infall velocity is the same at all radii at $t = 0$ (e.g., see eq. [6]) and predict constant mass accretion rates for $t > 0$. However, a similarity solution is only valid in a relatively small inner region (where boundary effects are not important), and numerical simulations show that the infall velocity is *not* spatially constant as protostar formation is reached. The MHD simulations of core collapse provide a natural reason; the inner solution has to match onto an outer solution of slow (ambipolar-diffusion regulated) infall through a subcritical envelope. Hence, the magnitude of the infall velocity peaks in the inner core and decreases outward toward the core boundary. The hydrodynamic simulations of Hunter (1977) and Foster & Chevalier (1993), which also had outwardly decreasing speeds due to the presence of a numerical boundary and followed the evolution past $t = 0$, found that $\dot{M}$ achieved a peak value just after $t = 0$ and subsequently decreased as mass shells with successively smaller infall speeds at $t = 0$ reached the protostar. We expect the same pattern in the realistic case with magnetic fields, although quantitative estimates of $\dot{M}$ for $t > 0$ in a supercritical core must await detailed calculations.

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