

Angular Momentum of Cores and Envelopes, and Magnetic Braking

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Abstract.

We study the effect of magnetic braking on the ambipolar-diffusion initiated formation and contraction of protostellar cores in axisymmetric, disklike, model molecular clouds. Magnetic braking rapidly enforces near corotation of the cloud with the background medium during the initial, ambipolar-diffusion controlled, quasistatic contraction. After a magnetically (and thermally) supercritical core forms, rapid contraction of the core ensues, and a power-law angular velocity profile is established within it, while the magnetically supported envelope continues to corotate with the background. The evolution of the central angular velocity is characterized by three distinct stages: (1) an exponential decrease due to effective magnetic braking; (2) a constant angular velocity phase, which lasts until a supercritical core forms; and (3) a constant angular momentum phase. Predictions are made for the angular momentum, mass, structure, size, etc. of protostellar cores.

Even in situations where angular momentum is conserved, we find that the centrifugal support does not become significant for mass shells in the central region of a self-gravitating, nonhomologously contracting thin disk; the gravitational field increases just as rapidly as the centrifugal acceleration ($1/r^3$).

1. Introduction

Recent observational studies of molecular clouds reveal a general picture of cool ($T \sim 10$ K), dense (density $\gtrsim 10^4$ cm $^{-3}$, size ~ 0.1 pc) condensations forming within larger (density $\sim 10^2 - 10^3$ cm $^{-3}$) molecular cloud envelopes (e.g., Myers & Benson 1983). These dense cores are often associated with young stars. Though the clouds as a whole are significantly thermally supercritical, there is now considerable evidence that they can be supported by magnetic fields. Optical polarization maps (e.g., Heyer et al. 1987, Goodman et al. 1990) reveal large scale ordered magnetic field structures in molecular clouds, and measurements of the field strength via the OH Zeeman effect in the B1 cloud in Perseus show that $|B| \approx 20\mu\text{G}$ (Crutcher et al. 1993). The recent survey by Goodman et al. (1993) found that most dense cores do have a statistically significant velocity gradient, ranging in magnitude from 0.3 to 4 km s $^{-1}$ pc $^{-1}$. Together, these observations show that the inferred rotational kinetic energy of dense cores is

much smaller than the magnitude of either the gravitational potential energy or the magnetic energy, which are comparable.

The observational results described above are in general agreement with the picture put forth by Mouschovias (1976, 1977, 1978, 1979) that relatively weak magnetic fields can support molecular clouds, within which the process of ambipolar diffusion (the relative drift of neutrals and plasma) allows the formation of cores. Magnetic braking was also shown to be a means to resolve the angular momentum problem of star formation. The angular momentum problem can be described as follows. Whereas a $1 M_{\odot}$ chunk of the interstellar medium ($n \sim 1 \text{ cm}^{-3}$) rotating at the local rate of galactic differential rotation ($\sim 10^{-15} \text{ rad s}^{-1}$) has a specific angular momentum $J/M \sim 10^{22} \text{ cm}^2 \text{ s}^{-1}$, a $1 M_{\odot}$ dense ($\sim 10^4 \text{ cm}^{-3}$) core with $\Omega \sim 1 \text{ km s}^{-1} \text{ pc}^{-1}$ has a $J/M \sim 10^{21} \text{ cm}^2 \text{ s}^{-1}$. Wide binary systems can have a J/M comparable to dense cores, but even a relatively wide binary system with a 100 yr period has a $J/M \sim 10^{20} \text{ cm}^2 \text{ s}^{-1}$. Finally, the present day solar system has a J/M of only $\sim 10^{18} \text{ cm}^2 \text{ s}^{-1}$.

In this paper, we present some results from the first study of both the quasi-static and dynamic phases of core formation and contraction in axisymmetric, isothermal, rotating, magnetic molecular clouds (Basu & Mouschovias 1994a). The calculations accurately determine the time-dependent angular momentum structure within the cloud and associated core. Determination of the amount of angular momentum that can ultimately find its way into a protostellar system is a key step in the formulation of a theory of star formation. For a discussion of the nonrotating case, see papers by Mouschovias and Ciolek in this volume.

2. Core Formation and Contraction in Rotating, Magnetic Clouds

2.1. Model Cloud

We consider axisymmetric, isothermal clouds embedded in an external medium of constant density and pressure. The cloud is threaded by a magnetic field which, far from the cloud, is uniform and directed along the axis of symmetry, which is the z -axis of a cylindrical polar coordinate system (r, φ, z) . The rotation axis is also taken to coincide with the z -axis. We take advantage of the recent results of Fiedler & Mouschovias (1993) (hereafter FM93), who find that a typical cloud rapidly relaxes along field lines until thermal-pressure forces balance gravity. Therefore, our clouds are modeled as thin disks, with force balance along the field lines from the outset. Specification of the initial distribution of mass-to-flux ratio M/Φ_B versus mass M (a monotonically decreasing function) and specific angular momentum J/M versus M (a linearly increasing function) allows a solution of the magnetohydrostatic equations to obtain an exact initial equilibrium state. These states are then allowed to evolve due to the onset of magnetic braking and ambipolar diffusion.

The model cloud presented here has the “standard” values of the dimensionless free parameters (see Basu & Mouschovias for a complete discussion of all free parameters in the problem). This magnetically subcritical cloud has an initial central mass-to-flux ratio in units of the critical value $\mu_{c0} = 0.32$. The full two fluid (ions and neutrals) MHD equations with self-gravity are solved numerically on a moving grid.

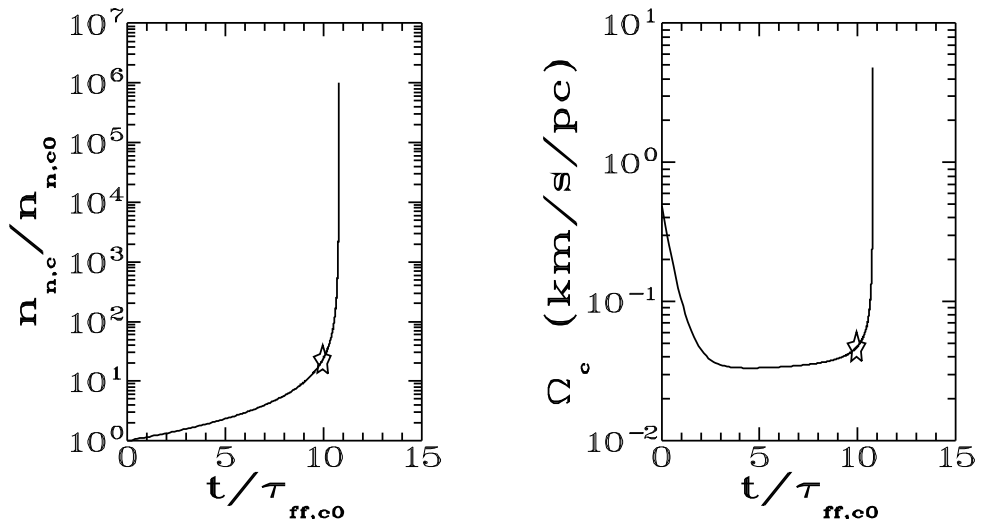


Figure 1. (a) Central density enhancement $n_{n,c}/n_{n,c0}$ and (b) central angular velocity Ω_c , versus $t/\tau_{\text{ff},c0}$.

2.2. Evolution of Central Values

Figure 1a shows the central neutral density of the cloud $n_{n,c}$, normalized to its initial central value $n_{n,c0} = 3.12 \times 10^3 \text{ cm}^{-3}$, as a function of time t , normalized to the initial central free-fall time $\tau_{\text{ff},c0} = 1.48 \times 10^6 \text{ yr}$. The evolution is followed through six orders of magnitude enhancement in central density, at which point the assumption of isothermality begins to break down. As found in nonrotating models (FM93, Ciolek & Mouschovias 1994, hereafter CM94), the evolution is initially quasistatic, and occurs on the ambipolar-diffusion time scale. A star marks the time at which the central flux tube becomes magnetically critical; this occurs at $t = 9.9 \tau_{\text{ff},c0} = 1.47 \times 10^7 \text{ yr}$. The remainder of the evolution is rapid, and the remaining nearly five orders of magnitude central density enhancement take only 1.24 Myr. The evolution of the central angular velocity Ω_c in $\text{km s}^{-1} \text{ pc}^{-1}$ is shown in Figure 1b. It is seen that Ω_c evolves through three distinct stages. In the first stage, Ω_c is reduced from its initial value $\Omega_{c0} = 0.5 \text{ km s}^{-1} \text{ pc}^{-1}$ to near corotation with the background medium ($\Omega_b = 10^{-15} \text{ rad s}^{-1} = 0.031 \text{ km s}^{-1} \text{ pc}^{-1}$) on the initial magnetic braking time scale $\tau_{J,c0}$. In the second stage, which lasts for the remainder of the quasistatic phase, magnetic braking enforces corotation of the cloud with the background medium. Finally, after the core becomes critical, the contraction of the core is so rapid that the magnetic braking time far exceeds the contraction time ($\tau_{J,c} \gg \tau_{\sigma,c}$), and angular momentum is effectively conserved.

2.3. Evolution of Spatial Profiles

Figure 2a shows the number density of neutrals n_n versus radius at seven different times (see caption). Stars and open circles on some curves mark the supercritical core boundary and the size of the central half-thickness, respectively.

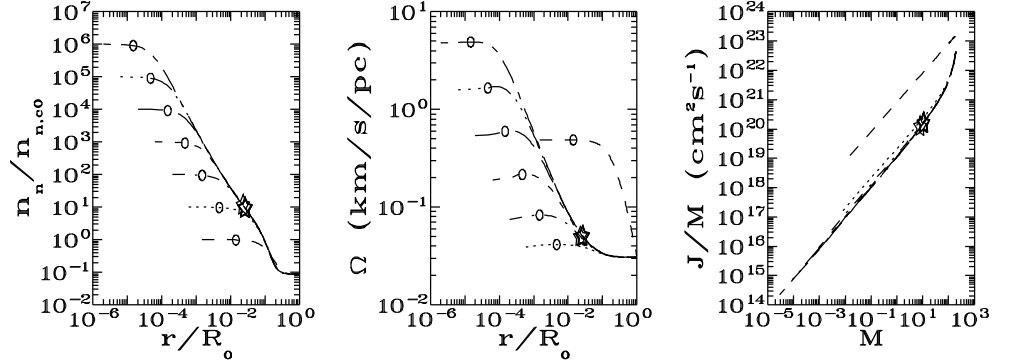


Figure 2. Spatial profiles at seven different times t_j ($j = 0, 1, \dots, 6$) chosen so as to have an enhancement of $n_{n,c}/n_{n,c0}$ by a factor 10^j ($j = 0, 1, \dots, 6$). (a) Neutral density $n_n/n_{n,c0}$ and (b) Angular velocity Ω , versus radius r , normalized to the initial cloud radius $R_0 = 5.76$ pc. (c) Specific angular momentum J/M versus M , in solar masses.

After ambipolar diffusion (through the inward drift of neutrals past plasma) leads to the formation of a region that is both thermally and magnetically supercritical, it contracts rapidly and creates a power-law density profile within it (barring an extremely compact, near-uniform density region near the cloud center), while the subcritical envelope very nearly retains its initial density profile. For this model, we find that the power-law index within the supercritical core has a mean value of -1.6 . The supercritical core has a radius of 0.14 pc at the end of the run and a mean density of $5 \times 10^4 \text{ cm}^{-3}$. Figure 2b shows the spatial profiles of the angular velocity Ω versus r/R_0 . It is clear that the cloud loses memory of its initial angular velocity profile between times t_0 (*dashed line*) and $t_1 = 13.38$ Myr (*dotted line*). The entire cloud is reduced to a near corotation with the background medium. As a supercritical core forms and separates out, a region develops in which angular momentum is conserved. During this time, the angular velocity profile has three spatial domains: (1) an outer region representing most of the cloud which is rotating at the background rate; within the supercritical core, (2) there exists a compact uniform- Ω region in its innermost part, and (3) a power-law region through the rest of the core. The angular velocity profile in the supercritical core closely follows the relation $\Omega \propto n_n^{1/2}$, a natural consequence of angular momentum conservation in a thin disk.

Figure 2c shows the specific angular momentum J/M versus the mass M , and illustrates the global effects of magnetic braking. There is a dramatic reduction in the specific angular momentum by the time $t = t_1$. This interval accounts for most of the quasistatic phase. During the short time interval between $t = t_2$ and $t = t_6$, there is very little change in the profile, due to effective angular momentum conservation in the core, and lack of sufficient evolution in the envelope. A linear relation of J/M versus M is established in the supercritical core. The supercritical core has a mass $10.9 M_\odot$, out of a total cloud mass of $191 M_\odot$, and a J/M of $1.6 \times 10^{20} \text{ cm}^2 \text{ s}^{-1}$. The matter in the supercritical

core reduces its J/M by nearly two orders of magnitude, to values comparable to those of wide binary systems.

The magnetic field components B_z and B_r evolve in a manner very similar to that in nonrotating magnetic models (FM93, CM94). The rotation of the cloud also introduces a component B_ϕ , which is kept small due to the efficiency of magnetic braking. Our calculations show a hierarchy of magnetic field component strengths: $|B_\phi|_{\max} \ll |B_r|_{\max} < |B_z|_{\max}$.

Although magnetic braking is ineffective in the supercritical core during the last nearly five orders of magnitude increase in central density, the centrifugal support does not rise for mass shells in this region. *The gravitational field on a mass shell in the central region increases just as rapidly ($1/r^3$) as the centrifugal acceleration.* This is a result of the thin disk geometry, where one does not obtain $g_r \propto 1/r^2$ as in the spherically symmetric case. This relationship can be justified by analytic arguments (Basu & Mouschovias 1994b).

3. Conclusion

We have summarized some results of a study of the effect of magnetic braking on the self-initiated (due to ambipolar diffusion) formation and contraction of molecular cloud cores. The angular velocity is held at the constant background value through most of the cloud, with an inner differentially rotating region in the supercritical core in which $\Omega \propto n_n^{1/2}$ is very nearly satisfied. The mean power-law index of the density in the magnetically supercritical core is -1.6 . The central angular velocity first suffers an exponential decrease towards the background rate, is held nearly constant at that rate through the quasistatic phase, and then increases about as rapidly as allowed by angular momentum conservation in the supercritical core. The gravitational field g_r increases so rapidly in the innermost region ($1/r^3$) as to keep centrifugal support negligible there. A parameter study of model clouds (Basu 1993) reveals that observationally reasonable values of the dimensionless free parameters can explain a range of core masses $3 - 30 M_\odot$ and angular momenta $J/M \sim 10^{19} - 10^{21} \text{ cm}^2 \text{ s}^{-1}$.

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