PHYSICS 2800 – 2^{nd} TERM
Introduction to Materials Science

Assignment 3

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You may assume the standard values $\mu_0 = 4\pi \times 10^{-7}$ kg-m/C^2 for the permeability of vacuum and $\mu_B = 9.27 \times 10^{-24}$ A-m^2 for the Bohr magneton.

1. Nickel is a ferromagnetic metal with density 8.90 g/cm^3. Given that Avogadro's number N_A = 6.02×10^{23} atoms/mol and the atomic weight of Ni is 58.7, calculate the number of atoms of Ni per cubic metre. If one atom of Ni has a magnetic moment of 0.6 Bohr magnetons, deduce the saturation magnetization M_s and the saturation magnetic induction B_s for Ni.

Answer:
The saturation magnetization is given by $M_s = 0.6\mu_B N$, where N is the number of atoms/m^3.

But $N = \dfrac{\rho N_A}{A_{Ni}}$, where ρ is the density and A_{Ni} is the atomic weight of Ni.

So $N = \dfrac{(8.90\times10^6 \text{ g/m}^3)(6.02\times10^{23} \text{ atoms/mol})}{58.7 \text{ g/mol}} = 9.13 \times 10^{28}$ atoms/m^3.

Then $M_s = 0.6\,(9.27 \times 10^{-24})(\,9.13 \times 10^{28}) = 5.1 \times 10^5$ A/m.
Finally (from the lecture notes),

$$B_S = \mu_0 M_S = (4\pi \times 10^{-7})(5.1 \times 10^5) = 0.64 \text{ Tesla}.$$

2. The magnetization within a bar of some metal alloy is 1.2×10^6 A/m when the H field is 200 A/m. Calculate (a) the magnetic susceptibility χ_m of this alloy, (b) the permeability μ, and (c) the magnetic induction B within the alloy. What type(s) of magnetism would you suggest as being displayed by this material (and explain why)?

Answer:

(a) We calculate the magnetic susceptibility within a bar of metal alloy when $M = 1.2 \text{ x } 10^6$ A/m and H =200 A/m. Solving for χ_m gives

$$\chi_{\text{m}} = \frac{M}{H} = \frac{1.2 \times 10^6 \, A/m}{200 \;\; A/m} = 6000$$

(b) In order to calculate the permeability we employ

$$\mu = \mu_{\text{r}}\,\mu_{\text{o}} = \mu_{\text{o}}\left(1+\chi_{\text{m}}\right)$$
$$= \left(1.257 \times 10^{-6} \text{ H/m}\right)(6000 + 1) = 7.54 \times 10^{-3} \text{ H/m}$$

(c) The magnetic induction B may then be found from

$$B = \mu H = (7.54\times10^{-3})(200) = 1.51\,\text{Tesla}$$

(d) This metal alloy would exhibit ferromagnetic behavior on the basis of its χ_m value (6000), which is considerably larger than typical χ_m values for diamagnetic and paramagnetic materials. Also M is nonzero and large. (It could also be ferrimagnetic).

3. Deduce the number of Bohr magnetons per atom of iron, given that the saturation magnetization $M_s = 1.70 \times 10^6$ A/m, that iron has a BCC crystal structure, and that that the edge length of the cubic unit cell is 0.287 nm.

Under certain conditions (e.g., in ultrathin films), iron can be grown with a FCC crystal structure. If M_s has approximately the same value as before and the number of Bohr magnetons per atom is the same as before, what does this allow you to say (if anything) about the edge length of the new cubic unit cell?

Answer:

Let n_B be the number of Bohr magnetons per atom, which we will calculate. By the definition of magnetization: $$n_B = \frac{M_S}{\mu_B N}$$

where N is the number of atoms per cubic meter. This found from the number of atoms per unit cell (2 for BCC) divided by the unit cell volume, so

$$N = \frac{2}{a^3}$$

with a being the BCC unit cell edge length (= 0.287 nm). Thus

$$n_B = \frac{M_S}{\mu_B N} = \frac{M_S a^3}{2\mu_B} = \frac{(1.70\times10^6)(0.287\times10^9)}{2(9.27\times10^{-24})} = 2.16 \quad \text{(in Bohr magnetons/atom).}$$

In the FCC case, we are told to take n_B and M_S to be the same, and therefore N must be the same. However, there are 4 atoms per unit cell in the FCC case, so to keep N the same we have

$$N = \frac{2}{a^3} = \frac{4}{(a_{FCC})^3}$$, where a_{FCC} is the new edge length.

Therefore $(a_{FCC})^3 = 2a^3$, so $a_{FCC} = 1.26a = 0.362$ nm

4. An iron bar magnet having a coercivity of 4000 A/m is to be demagnetized. If the bar is inserted within a cylindrical wire coil (a solenoid) 25 cm long and having 150 turns, what electric current is required to generate the necessary magnetic field?

Answer:

To demagnetize a magnet having a coercivity of 4000 A/m, an H field of 4000 A/m must be applied in a direction opposite to that of the magnetization. From the notes, the H field in a solenoid is: $H = NI/L$, where L is the length.

$$I = \frac{HL}{N} = \frac{(4000)(0.25)}{150} = 6.67 \text{ A}$$

5. At any temperature T less than the critical temperature T_C for a superconducting material, the critical field $H_C(T)$ depends approximately on temperature according to

$$H_C(T) = H_C(0)\,[1 - (T/T_C)^2]$$

where $H_C(0)$ is the critical field at 0 K. Use this to do the following calculations.

(a) Given that $T_C = 4.15$ K and $H_C(0) = 3.27 \times 10^4$ A/m for mercury, calculate the critical magnetic fields for superconductivity to exist in mercury at temperatures of 2.0 K, 3.0 K, and 5.0 K.

(b) To what temperature must mercury, initially at 10 K, be cooled in a magnetic field of 15,000 A/m for it to become superconducting?

Answer:

(a) At $T = 2$ K, $(T/T_C)^2 = 0.232$ which implies $H_C(T) = 2.51 \times 10^4$ A/m
At $T = 3$ K, $(T/T_C)^2 = 0.523$ which implies $H_C(T) = 1.56 \times 10^4$ A/m
At $T = 5$ K, T is above T_C and SC is impossible; $H_C(T)$ does not exist.

(b) Rearranging, $T = T_C\,[1 - \{H_C(T)/H_C(0)\}]^{1/2} = 4.15\,(1 - \{15000/32700\}]^{1/2}$
$= 3.05$ K

6. Consider the following superconducting elements:

Lead	$T_C = 7.19$ K	$H_C(0) = 6.39 \times 10^4$ A/m
Mercury	$T_C = 4.15$ K	$H_C(0) = 3.27 \times 10^4$ A/m
Tin	$T_C = 3.72$ K	$H_C(0) = 2.41 \times 10^4$ A/m

Assuming the same formula as in Question 5, deduce which of the above elements are superconducting at 2 K when placed in a magnetic field of 40,000 A/m.

Answer:

For Pb at 2 K, $(T/T_C)^2 = 0.077$ which implies $H_C(T) = 8.67 \times 10^4$ A/m.
So $H_C(T) > H$ and Pb is <u>SC</u> under these conditions.
For Hg at 2 K, $H_C(T) < H_C(0) < H$ and Hg is <u>not SC</u> under these conditions.
For Sn at 2 K, $H_C(T) < H_C(0) < H$ and Hg is <u>not SC</u> under these conditions.